

# Math 113 Homework 5

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There are four problems, due Wednesday, March 6.

1. Consider the quaternion group  $Q = \{1, -1, i, -i, j, -j, k, -k\}$  with the operation discussed in class (also see the course homepage for links).
  - (a) For each element of  $Q$ , find its order.
  - (b) Find all subgroups of  $Q$ . [Hint: there are six of them.]
  - (c) Which of the subgroups in part (a) are isomorphic to each other?
2. Consider the group  $Sym_3$ . List all six of its subgroups.
3. Let  $G = Sym_3$  and  $S = \{1, 2, 3, 4, 5, 6\}$ . Define an action of  $G$  on  $S$  as follows. The action corresponds to a homomorphism  $\phi: G \rightarrow \Sigma(S)$  sending  $(12) \in G$  to  $(12)(45) \in Sym_6 = \Sigma(S)$  and  $(123) \in G$  to  $(123) \in \Sigma(S)$ .
  - (a) Explain why  $\phi$  is uniquely specified by what it does to  $(12)$  and  $(123)$  [Hint: use HW 3 Question 5].
  - (b) Find  $\phi((23))$  and  $\phi((132))$ .
  - (c) For each element of  $S$ , find its stabilizer as a subgroup of  $G$ .
  - (d) List the orbits of the group action. How many orbits are there, and how many elements does each orbit have?
4.
  - (a) How many elements of  $Sym_5$  have order 2?
  - (b) How many have order 3?