

Math 113 Homework 2

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There are six problems, due Wednesday, February 13.

1. Show that $x^2 = 3$ has no solutions for rational numbers x . [Hint: write $x = a/b$, for a and b integers, then use what you know about unique prime factorization.]
2. We say that the *cancellation law* holds in $\mathbb{Z}/m\mathbb{Z}$ if for any $a, b, c \in \mathbb{Z}/m\mathbb{Z}$ such that $c \neq 0 \pmod{m}$, the equation $ac = bc$ implies $a = b$.
 - (a) Does the cancellation law hold in $\mathbb{Z}/5\mathbb{Z}$?
 - (b) Does the cancellation law hold in $\mathbb{Z}/21\mathbb{Z}$?

In either case, explain why, or give a counterexample.

3. On p.7 of the course notes ([P]), there are eight properties of addition and multiplication on \mathbb{Z} (four for addition, three for multiplication, and one about distributivity). Prove the following statements using only these eight properties:
 - (a) For any $a \in \mathbb{Z}$, we have $0 \times a = 0$.
 - (b) For any $a \in \mathbb{Z}$, we have $(-1) \times a = -a$.
4. In each of the following problems, prove your answer. Note that if an inverse exists, you don't have to find it; you just have to explain why it exists.
 - (a) Does 38 have a multiplicative inverse modulo 82?
 - (b) Does 51 have a multiplicative inverse modulo 82? Prove your answer. [Hint: if it has an inverse, you]
5. Let $S = \{a, b, c\}$ be a set with three elements.

(a) How many binary operations are there on the set S ?

(b) How many of these binary operations give S the structure of a group?

6. Let $G = \{a, b\}$, and define an operation $*$: $G \times G \rightarrow G$ by

$$a * a = a$$

$$a * b = b$$

$$b * a = b$$

$$b * b = b$$

Is G a group under this operation? Prove your answer.