

Math 113 Homework 2

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There are five problems, due Tuesday, September 17.

1. We say that the *cancellation law* holds in $\mathbb{Z}/m\mathbb{Z}$ if for any $a, b, c \in \mathbb{Z}/m\mathbb{Z}$ such that $c \neq 0 \pmod{m}$, the equation $ac = bc$ implies $a = b$.

- (a) Does the cancellation law hold in $\mathbb{Z}/5\mathbb{Z}$?
- (b) Does the cancellation law hold in $\mathbb{Z}/21\mathbb{Z}$?

In either case, explain why, or give a counterexample.

2. On p.14-15 of the course notes ([P]), there are eight properties of addition and multiplication on \mathbb{Z} (four for addition, three for multiplication, and one about distributivity). Prove the following statements using only these eight properties:

- (a) For any $a \in \mathbb{Z}$, we have $0 \times a = 0$.
- (b) For any $a \in \mathbb{Z}$, we have $(-1) \times a = -a$.

3. In each of the following problems, prove your answer. Note that if an inverse exists, you don't have to find it; you just have to explain why it exists.

- (a) Does 38 have a multiplicative inverse modulo 82?
- (b) Does 51 have a multiplicative inverse modulo 82? Prove your answer.
[Hint: if it has an inverse, you]

4. Let $S = \{a, b, c\}$ be a set with three elements.

- (a) How many binary operations are there on the set S ? (Hint: recall, very carefully, what a binary operation is.)
- (b) How many of these binary operations give S the structure of a group?

5. Let $G = \{a, b\}$, and define an operation $*$: $G \times G \rightarrow G$ by

$$a * a = a$$

$$a * b = b$$

$$b * a = b$$

$$b * b = b$$

Is G a group under this operation? Prove your answer.