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Spring 2006, Math 104
Second Midterm

24 March, 2006
3:10-4:00 PM

1. (32 points, 8 points each.) Complete the following definitions. You may use, without defining them, terms or symbols that Rudin defines before he defines the word or symbol asked for. Your definitions do not have to have exactly the same wording as those in Rudin, but for full credit they should be clear, and mean the same thing as his.

(a) A subset E of a metric space X is said to be *compact* if

(b) A series $\Sigma_{n=1}^{\infty} a_n$ is called *absolutely convergent* if

(c) If f is a function from a segment $(a,b) \subseteq R$ to a metric space X, and if $u \in (a,b)$ and $x \in X$, then we write $f(u+) = x$ if

(d) If f is a bounded function on an interval $[a,b]$, then $\overline{\int} f dx$ denotes $\inf U(P,f)$. Here the $\inf$ is taken over all ______________, and $U(P,f)$ denotes $\Sigma_{i=1}^{n} M_i \Delta x_i$, where for each i, $M_i =$ ______________, and $\Delta x_i =$ ______________.

2. (32 points, 8 points each.) For each of the items listed below, either *give an example* with the properties stated, or give a brief reason why *no such example exists.*

If you give an example, you do *not* have to prove that it has the property stated; however, your examples should be specific; i.e., even if there are many objects of a given sort, you should name a particular one. If you give a reason why no example exists, don't worry about giving reasons for your reasons; a simple statement will suffice. (a) A finite subcovering of the covering $\{(x-0.01, x+0.01) \mid x \in [0,1]\}$ of the interval $[0,1]$.

(b) A sequence (s_n) of points of the segment $(0,1)$, such that $\lim_{n\to\infty} s_n$ exists in R, but does not belong to $(0,1)$.

(c) A metric space X, and a Cauchy sequence in X which does not converge in X.

(d) A differentiable function $f: (-1,1) \to R$ such that $f'(x) \leq -1$ for all $x < 0$, and $f'(x) \geq 1$ for all $x > 0$.

3. (18 points) If X, Y and Z are metric spaces, and if $f: X \to Y$ and $g: Y \to Z$ are uniformly continuous functions, and we define $h: X \to Z$ by $h(x) = g(f(x))$, show that the function h is also uniformly continuous.

4. (18 points) Suppose f is a differentiable function on a segment (a,b) $(a, b \in R)$, and is unbounded. (I.e., there is no $M > 0$ such that $|f(x)| < M$ for all $x \in (a,b)$.) Prove that f' is also unbounded.