

Department of Mathematics, University of California, Berkeley

Math 1B

Alan Weinstein, Spring 2002

First Midterm Exam, Thursday, February 21, 2002

Instructions. Be sure to write on the front cover of your blue book: (1) your name, (2) your Student ID Number, (3) your GSI's name (Tathagata Basak, Tameka Carter, Alex Diesl, Clifton Ealy, Peter Gerdes, John Goodrick, Matt Harvey, George Kirkup, Andreas Liu, Rob Myers, or Kei Nakamura).

Read the problems very carefully to be sure that you understand the statements. Show all your work as clearly as possible, and circle each final answer to each problem. **When doing a computation, don't put an "=" sign between things which are not equal. When giving explanations, write complete sentences. Remember: if we can't read it, we can't grade it.**

1 . [20 points] Evaluate each of the following.

(A)

$$\int e^{\sqrt{x}} dx$$

(B)

$$\int_{-\pi/3}^0 \sin^2 x dx + \int_0^{\pi/3} \cos^2 x dx$$

(C)

$$\int_0^{\pi/2} \sin 2x \sin x dx$$

(D)

$$\int \frac{x^2 + 2}{x^2 - x} dx$$

2 . [10 points] For which values of p is the integral

$$\int_0^{\pi} \frac{\sin x}{x^p} dx$$

improper? For which of these values of p is it convergent? You must justify your answers.

3 . [15 points] Suppose that a thin wire of uniform linear density λ (i.e. the mass of any segment of the wire is λ times its arc length) is represented by the graph of a function $y = f(x)$ defined in the interval $[a, b]$. Putting together what you know about arc length and centers of mass, write a formula for the mass m of the wire, and a pair of integral formulas for the coordinates (\bar{x}, \bar{y}) of the center of mass of the wire. The integrands in the formulas should be expressed in terms of $f(x)$ and, possibly, its derivative(s).

You must give some justification for your formulas (a limit argument or a "differential" argument), but it does not have to be a formal proof.