Second Midterm Exam—60 points

1 *(6 points each)*. Find the sums of the following series:

a. $\sum_{n=1}^{\infty}\left[\ln\left(\frac{n+1}{n}\right)-\ln\left(\frac{n+2}{n+1}\right)\right]$.

b. $\sum_{n=3}^{\infty}\left(3\left(\frac{3}{4}\right)^{n}-4\left(-\frac{1}{2}\right)^{n+1}\right)$.

2 *(7 points each)*. Determine whether each series is divergent, conditionally convergent, or absolutely convergent:

a. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(n+1)!-n!}{(n+2)!-(n+1)!}$

b. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^3}\tan\left(\frac{\pi}{4}+\frac{1}{n}\right)$.

3 *(5 points each)*. Decide whether the following infinite series converge or diverge:

a. $\sum_{n=1}^{\infty}\left(1-\cos\frac{1}{n}\right)$.

b. $\sum_{n=1}^{\infty}\left(1-\frac{2}{n}\right)^{n^2}$.

4a *(6 points)*. Consider the curve with polar equation $r = 1+2\cos(\theta)$. Express in terms of definite integrals the area inside the small loop. [The problem was accompanied by a small graph of the curve, produced by Mathematica.]

4b *(6 points)*. Find the slope of the line tangent to the polar curve $r = 1+2\cos(\theta)$ at the point with $\theta = \pi/2$.

5a *(5 points)*. Express in terms of definite integrals the length of the entire curve $r = 1+2\cos(\theta)$ (both loops).

5b *(7 points)*. Suppose that $f(x) = \sum_{n=0}^{\infty} a_n x^n$ for all x such that $|x| \le \frac{1}{2}$. What function has Taylor series $\sum_{n=2}^{\infty} n^2 a_n x^n$?

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