Name: ....

University of California, Math 142 Final Exam, 15 May, 2006

Prof. R. Kirby

1. Consider the covering space X of the torus T^2 corresponding to the subgroup $pZ \times qZ$ of π_1. Compute the homomorphism $f_* : H_k(X, Z) \to H_k(T^2, Z)$ for $k = 0, 1, 2$.

2. The rationals Q are a subgroup of the additive group of the reals R. Is R/Q Hausdorff? Prove your answer.

3. Suppose a CW complex X is constructed with a finite number of cells. Prove that X is compact

4. Describe the lens space $L(p, q)$ as surgery on a knot, and as a CW complex. Then calculate its homology groups with Z coefficients.

5. Calculate $\pi_1(X \times Y, (x_0, y_0))$ for two path connected topological spaces X, x_0 and Y, y_0.

6. Show that $\pi_1(X * Y, (x_0, 1/2, y_0)) = 0$ for two path connected spaces X and Y, and their join $X * Y$.

7. Compute the Euler characteristic of P^n, n-dimensional projective space.

8. Describe the closed, orientable surfact of genus g as a CW complex. Then compute its homology groups with Z coefficients.

9. Compute the first homology group with Z coefficients of the topological space which is the complement of the Borromean rings.

10. Given a topological space X, x_0 with base point, and given a subgroup H of $\pi_1(X, x_0)$, define the covering space of X corresponding to H. How many points in a fiber? What can you say about the covering transformations?