## Department of Mathematics, University of California, Berkeley

# Math 1B

### Alan Weinstein, Spring 2002

### Second Midterm Exam, Thursday, April 11, 2002

**Instructions.** Be sure to write on the front cover of your blue book: (1) your name, (2) your Student ID Number, (3) your GSI's name (Tathagata Basak, Tameka Carter, Alex Diesl, Clifton Ealy, Peter Gerdes, John Goodrick, Matt Harvey, George Kirkup, Andreas Liu, Rob Myers, or Kei Nakamura).

Read the problems very carefully to be sure that you understand the statements. Show all your work as clearly as possible, and circle each final answer to each problem. **When doing a computation, don't put an "=" sign between things which are not equal. When giving explanations, write complete sentences. Remember: if we can't read it, we can't grade it.**

**1.** [8 points] Find the general solution of the differential equation $dy/dx = x^2 y^2$, and find the particular solution for which $y = 1$ when $x = 1$.

**2.** [5 points] For which values of $p$ is the series $\sum_{n=1}^{\infty} \sin(1/n) n^p$ convergent?.

**3.** [5 points] Show that absolute value $|\sin x - x - x^3/6|$ is less than $1/100$ for all $x$ in the interval $(-1, 1)$.

**4.** [9 points] For each of the following statements, tell whether it is true or false, and give a justification for your answer. In particular, the falsity of an "if/then" statement should be justified by a counterexample. (A correct T or F gets no credit without a correct justification.)
(A) If the sequence $\{a_n\}$ converges to $L$, then $a_{100}$ is closer to $L$ than $a_{99}$ is.
(B) If the series $\sum_{n=1}^{\infty} a_n$ is convergent, then the sequence $\{a_n\}$ is also convergent.
(C) If $a_n \to 0$ as $n \to \infty$, then the series $\sum_{n=1}^{\infty} (-1)^n a_n$ is convergent.

**5.** [11 points] The velocity $v = \frac{dx}{dt}$ of a vehicle on a experimental highway is controlled by signals transmitted from the roadway so that, when the vehicle is at the position $x$ miles from the beginning of the highway, its velocity is $20x + 10$ miles per hour.
(A) Find the position of the vehicle as a function of $t$ if it starts at $x = 0$ when $t = 0$.
(B) How long does it take the vehicle to travel from $x = 0$ to $x = 3$ if it starts at $x = 0$ when $t = 0$?
(C) How long does it take the vehicle to travel from $x = 0$ to $x = 3$ if it starts at $x = 0$ when $t = 1$?

**6.** [7 points] Find an explicit formula for the function of $x$ represented by the power series $\sum_{n=0}^{\infty} (3^n + 1) x^n$. What is the radius of convergence of the series?