Math 104, Midterm 1, 200809221 1–12, O. HALD

① Let $s_{n+1} = \dfrac{1+2s_n}{3+2s_n}$ with $s_1 = 1$.

(a) Show that $s_n > 0$ for $n \geq 1$.

(b) Let $s > 0$ satisfy $s = \dfrac{1+2s}{3+2s}$. Prove $\lim_{n\to\infty} s_n = s$.

② Assume $|t_n - 4| < \dfrac{4^4}{\sqrt[4]{n}}$. For every $\varepsilon > 0$ find $N$

such that $\left|\dfrac{1}{t_n} - \dfrac{1}{4}\right| < \varepsilon$ for $n > N$.

③ Let $s_n = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots + \frac{(-1)^{n+1}}{n}$.

(a) Show that $s_1 > s_3 > \cdots > s_{2n-1} > s_{2n+1} > \cdots > 0$.

Hint: $s_5 = 1 - (\frac{1}{2} - \frac{1}{3}) - (\frac{1}{4} - \frac{1}{5}) = (1 - \frac{1}{2}) + (\frac{1}{3} - \frac{1}{4}) + \frac{1}{5}$

(b) Is $s = \inf_{n \geq 1} (s_{2n-1})$ finite?

(c) Show that $\lim_{n\to\infty} (s_{n+1} - s_n) = 0$.

(d) Show that $\lim_{n\to\infty} s_n = s$.