MATH 110-6 F'03 KRAUSS

Midterm II

1. Let P_2 and P_3 be the vector spaces of polynomials over $\mathbb{R}$ of degree at most 2 and 3, respectively. Consider the bases $\mathbb{B}_2 \equiv \{2, -t, t^2\}$ of P_2 and $\mathbb{B}_3 \equiv \{1, 2t, t^2, -t^3\}$ of P_3.
   (i) Prove that $T: P_3 \to P_2$, $T(u) \equiv \frac{d^2}{dt^2}u - \frac{du}{dt}$ is a linear transformation.
   (ii) Find $[T]_{\mathbb{B}_3, \mathbb{B}_2}$.
   (iii) Find a basis for the orthogonal complement to the row space of $[T]_{\mathbb{B}_3, \mathbb{B}_2}$ under the standard inner product.

2. Consider the vector space $V \equiv \mathbb{R}^3$ with the standard inner product and let $x \equiv [1\ 0\ 1]^T$, $y \equiv [0\ 1\ 1]^T$, $z \equiv [1\ 1\ 0]^T$ be vectors of V.
   (i) Find the vector w of $W \equiv \operatorname{span}\{x, y\}$ closest to z under the induced distance on V.
   (ii) What is the error $\|z - w\|$?

3. (i) Prove that $\operatorname{tr} A = \operatorname{tr} B$ when $A = M^{-1}BM$ for some M, where A, B, M are square matrices and $\operatorname{tr} A = \sum_i A_{ii}$.
   (ii) Deduce that if A has n linearly independent eigenvectors, then $\operatorname{tr} A$ is the sum of the associated eigenvalues.
   (iii) Explain why in case (ii) one has $\det(A)$ is the product of the associated eigenvalues.