MATH 104 SECOND MIDTERM

April 2, 2003 H. Wu

Your Name: ______________________

1. (20%) Suppose S is a set of real numbers bounded above and $x_0 = \sup S$. Suppose also that $x_0 \notin S$. Prove that there exists a sequence $\{x_n\}$ in S so that $x_n < x_{n+1}$ for every $n \in \mathbb{N}$ and $x_n \to x_0$.

2. (10%) Suppose $\{t_n\}$ is a sequence converging to $T \in \mathbb{R}$, and $t_n \leq A$ for all $n \in \mathbb{N}$. Prove that $T \leq A$.

3. (20%) State and prove the Root Test.

4. (25%) Let $\{s_n\}$ and $\{t_n\}$ be bounded sequences. Prove that if $\forall n$, $s_n \geq 0$ and $t_n \geq 0$,

$$\limsup s_n t_n \leq (\limsup s_n)(\limsup t_n)$$

5. (25%) Suppose that $\{s_n\}$ is a sequence so that $|s_{n+1} - s_n| < \dfrac{1}{n^2}$ for all $n \in \mathbb{N}$. Is $\{s_n\}$ convergent? Why?