SEMINAR 6

MATTER IS WAVE
(De Broglie 1924)

WAVE IS MATTER
(Einstein 1905)

(sounds like
black is white)

\[ h = 6.626 \times 10^{-27} \, \text{g cm}^2 \text{s} \]

Planck constant

\[ \text{guess the units?} \]
\[ p = \text{momentum of a particle} \]

\[ \lambda = \text{wavelength of a wave} \]

\[ \lambda = \frac{\hbar}{p} \]

or

\[ p = \hbar k \]

\[ k = \frac{2\pi}{\lambda} \]

Wave number

\[ \hbar = \frac{\hbar}{2\pi} \]
Look for pictures of fun wave functions on-line...

A wave is described by a wave function

\[ \psi(x) = e^{i k x} \]

or a vector

\[ \psi = \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_n \end{pmatrix} \]

where \( \psi_j = e \)
To interpret $\gamma$

Physically we need so probability theory...

\[ 0 \leq x \leq 2\pi \]

To mean $x$ we assign its probability

\[ 0 \leq p(x) \leq 1 \]

**Rule**

\[ \sum_{x} p(x) = 1 \]
e.g. \( x = 0, \pi, \frac{3\pi}{4} \)

\[ p(0) = \frac{1}{2}, \quad p(\pi) = \frac{1}{3}, \quad p\left(\frac{3\pi}{4}\right) = \frac{1}{6} \]

Mean or Expected value

\[ \langle x \rangle = \sum x \cdot p(x) \]
In the example

\[ \langle x \rangle = 0 \cdot \frac{1}{2} + \pi \cdot \frac{1}{3} + \frac{3\pi}{4} \cdot \frac{1}{6} \]

\[ = \frac{11}{24} \pi \]

\[ p(0) = \text{prob. particle at } x = 0 \]

\[ p(\pi) = -11 \quad x = \pi \]

\[ p\left(\frac{3\pi}{4}\right) = -11 \quad x = \frac{3\pi}{4} \]
We can take expected value of any function of $x$:

$$\langle f(x) \rangle = \sum_x f(x) \rho(x)$$

E.g.,

$$\langle x^2 \rangle = \sum_x x^2 \rho(x)$$
Measure of Uncertainty
\[ \sqrt{\langle (x - \langle x \rangle)^2 \rangle} \]

Calculate
\[ \langle x - \langle x \rangle \rangle^2 = \sum_x (x - \langle x \rangle)^2 \rho(x) \]
\[ = \sum_x (x^2 - 2x\langle x \rangle + \langle x \rangle^2) \rho(x) \]
\[ = \sum_x x^2 \rho(x) - 2\langle x \rangle \sum_x x \rho(x) + \langle x \rangle^2 \sum_x \rho(x) \]
\[ = \langle x^2 \rangle - 2\langle x \rangle^2 + \langle x \rangle^2 \]
\[ = \langle x^2 \rangle - \langle x \rangle^2 \]