ERRATA TO “SEMICLASSICAL ANALYSIS” BY M ZWORSKI

Many thanks to Plamen Stefanov, Fréderic Klopp, Long Jin and Minjae Lee for pointing out errors and misprints, and for suggesting solutions.

- page 43, the displayed formula of step 4 of the proof should read
  \[ h^k J(0, P^k u) = 2\pi P^k u(0) = 2\pi (h\epsilon/2i)^k u^{(2k)}(0). \]

- page 47, Lemma 3.14: The constant \( C \) depends also on \( \varphi \).

- page 57, EXAMPLES: quantization is applied formally here as the definitions have been so far given only for symbols in \( \mathcal{S} \); Theorem 4.1 below justifies the use of more general class of symbols.

- page 58, step 2 should read: “The kernel of \( \text{Op}_t(a)\) is \( K^*_t(x, y) := K_t(y, x) \), which is the kernel of \( \text{Op}_1-t(\bar{a})\)”

- page 60, line 5 from the bottom: \( c_j(\frac{x+y}{2}, \xi) \) should be \( c_j(\frac{x+y}{2}) \) (\( c_j \) is independent of \( \xi \)).

- page 78: in Step 2 the sentence should be “… the method of stationary phase and Theorem 4.8 give…”

- page 80, (4.4.18) should be
  \[ a \# b = ab + \frac{h}{2i} \{ a, b \} + O_S(h^{1-2\delta}), \]
  that is, \( i/2h \) should be \( h/2i \).

- page 111: the last line in the second displayed formula in Step 2 should be
  \[ \geq \frac{1}{2} (\text{Im } \tau)^2 \| u \|_{L^2}^2. \]
  (a square is missing in the book)

- page 116: the estimate (5.3.31) is incorrect as stated. To obtain the correct version, we return to (5.3.30) and note that pole of \( P(\tau)^{-1} \) at 0 is simple and the image of the residue, \( A \), is \( \mathbb{C} \) (constant functions). Hence,
  \[ \ddot{u}_1(\tau) = \ddot{u}_0(\tau) + \ddot{u}_2(\tau), \]
  \[ \ddot{u}_0(\tau) := c_1 \int_{\mathbb{T}^n} \ddot{g}_1(\tau, x) dx/\tau, \quad \ddot{u}_2(\tau) := (P(\tau)^{-1} - A/\tau) \ddot{g}_1(\tau), \]
where $Q(\tau) := P(\tau)^{-1} - A/\tau$ satisfies the same estimates as $P(\tau)^{-1}$ but is holomorphic near 0. We note that $u_0 = c_0$ for $t > 0$.

We can now replace $u_1$ with $u_2$ as the constant term does not affect the energy estimate. Another way to look at this is changing the initial conditions from $u|_{t=0} = 0$, $\partial_t u|_{t=0} = f$, to $u|_{t=0} = -c_0$, $\partial_t u|_{t=0} = f$. This does not change energy $E(t)$.

In Step 3 we now have, for $u_2$, which has the same energy of $u_1$,

\[
\|e^{\beta t}u_2\|_{L^2(\mathbb{R}^n; H^1)} = (2\pi)^{-\frac{1}{2}} \|e^{\beta t}u_2\|_{L^2(\mathbb{R}; H^1)} \\
= (2\pi)^{-\frac{1}{2}} \|\hat{u}_2(\cdot - i\beta)\|_{L^2(\mathbb{R}; H^1)} \\
= (2\pi)^{-\frac{1}{2}} \|Q(\cdot - i\beta)^{-1}\hat{g}_1(\cdot - i\beta)\|_{L^2(\mathbb{R}; H^1)} \\
\leq C \|\hat{g}_1(\cdot - i\beta)\|_{L^2(\mathbb{R}; L^2)}.
\]

The remainder of the proof is the same, with $u - c_0$ in place of $u$.

- page 129, Theorem 6.7: the statement about the constant $h_0$ should be made before (i) as $0 < h < h_0$ is also required for (ii) and (iii). (The statements are actually true for all values of $h$ but that is not our concern.)
- page 130, line 5 of step 2: $K(-i,h)$ should be $K(i,h)$.
- page 135, line 3 from the bottom should read

\[
\|b^w(x,hD)\| \leq \lambda + \frac{3\epsilon}{4},
\]

for $0 < h < h(\epsilon)$. This follows, for instance from Theorem 4.30 applied to $(\lambda + \frac{3\epsilon}{4})^2 - (b^w(x,hD))^*b^w(x,hD)$ (see also Theorem 13.13).
- page 188, Remark: the second sentence should read “According to Theorem 8.10, if $a \in h^kS(m)$ for some $k \in \mathbb{R}$ and some order function $m$, then $T = a^w(x,hD)$ is tempered.
- page 190: (8.4.7) should be

\[
\text{WF}_h \left( \exp \left( i \langle x, \omega \rangle / h^\alpha \right) \right) = \begin{cases} 
\mathbb{R}^n \times \{0\}, & \alpha < 1, \\
\mathbb{R}^n \times \{\omega\}, & \alpha = 1, \\
\emptyset & \alpha > 1.
\end{cases}
\]

that is, the division by $h^\alpha$ should be in the exponent.
- page 214, in formula (9.3.26), $C$ is missing on the right; the expression should be:

\[
\langle a^w(x,hD)u,u \rangle \geq -Ch\|u\|^2_{H^m_{h^{m-1}}}, \quad u \in \mathcal{S}.
\]
- page 214, last line: $\text{spt } \tilde{\chi}_j$ should be $\text{spt } \tilde{\chi}_j$. 


• page 276: the first two references to [H2] should be to [H3] and the next two to [H4].

• page 277, line 7: “canonical transformation” means “symplectic transformation”.

• page 278, Thm 12.4: The constant $C$ in (12.2.3) depends on $P$ and the neighbourhood containing WF($u$).

• page 279-280: In Theorem 12.5 and in its proof (which determines the flow) needs to be replaced by $p_0$. Since the proof reduces the general case to the normal form we apply the Jacobi Theorem 2.10 (the flow of $\xi_1$ becomes that of $p_0$) to make the conclusion about the general case.

• page 323: the proof in Step 2 is incorrect. First, replace (13.5.7) with

$$\|b^w(x, hD)\|^2 \geq \langle M_{|q|^2} T_u, T_u \rangle_{L^2_{\Phi}} - C_0 h, \quad \|u\|_{L^2} = 1.$$  

and assume that $\Phi = |z|^2/2$. If the supremum of $|q|^2$ is achieved at, say, $z = 0$ then $\partial |q|^2(0) = 0$, and for $v = (2\pi h)^{-n/2}$

$$\langle M_{|q|^2} v, v \rangle_{L^2_{\Phi}} = \frac{1}{(2\pi h)^n} \int_{C^n} |q(z)|^2 e^{-|z|^2/h} \, dm(z)$$

$$= \frac{1}{(2\pi h)^n} \int_{C^n} (|q(0)|^2 + O(|z|^2)) e^{-|z|^2/h} \, dm(z) = |q(0)|^2 + O(h).$$

If the supremum is not attained then $|q(z_n)|^2 \to \sup |q|^2$, where $z_n \to \infty$. Since $|\partial^2 q| \leq C$, we have $\partial |q(z_n)|^2 \to 0$. We then choose $n$ large enough so that $|q(z_n)|^2 \geq \sup |q|^2 - h$, and $|\partial q(z_n)| \leq h$, translate $z_n$ to 0, and use the previous argument (or, directly, use $v(z) = (2/\pi h)^{n/2} (\det \Phi_z)^{1/2} e^{\Psi(z, \bar{z}_n)/h}$ as a test function).

• page 347, first displayed formula in §14.2.2: $S(\mathbb{R}^n)$ should be $S(\mathbb{R}^{2n})$. 