

ERRATA TO “SEMICLASSICAL ANALYSIS” BY M ZWORSKI

Many thanks to **Plamen Stefanov, Frédéric Klopp, Long Jin and Minjae Lee and Semyon Dyatlov** for pointing out errors and misprints, and for suggesting solutions.

- page 32, in step 1 of the proof of Lemma 3.3, to conclude that $\varphi_j \in \mathcal{S}(\mathbb{R}^n)$ we need to assume that $\varphi(y) = 0$; but that is the case which (3.1.14) is used in the next step.
- page 43, the displayed formula of step 4 of the proof should read

$$h^k J(0, P^k u) = 2\pi P^k u(0) = 2\pi (h\epsilon/2i)^k u^{(2k)}(0).$$

- page 47, Lemma 3.14: The constant C depends also on φ .
- page 53: in Remark (ii) the integration should be over \mathbb{R}^k and not \mathbb{R}^m .
- page 57, EXAMPLES: quantization is applied formally here as the definitions have been so far given only for symbols in \mathcal{S} ; Theorem 4.1 below justifies the use of more general class of symbols.
- page 58, step 2 should read: “The kernel of $\text{Op}_t(a)^*$ is $K_t^*(x, y) := \overline{K}_t(y, x)$, which is the kernel of $\text{Op}_{1-t}(\bar{a})$ ”
- page 60, line 5 from the bottom: $c_j(\frac{x+y}{2}, \xi)$ should be $c_j(\frac{x+y}{2})$ (c_j is independent of ξ).
- page 78: in Step 2 the sentence should be “... the method of stationary phase and Theorem 4.8 give...”
- page 80, (4.4.18) should be

$$a\#b = ab + \frac{h}{2i}\{a, b\} + O_{S_\delta(m_1 m_2)}(h^{1-2\delta}),$$

that is, $i/2h$ should be $h/2i$.

- page 111: the last line in the second displayed formula in Step 2 should be

$$\geq \frac{1}{2}(\text{Im } \tau)^2 \|u\|_{L^2}^2.$$

(a square is missing in the book)

- page 116: the estimate (5.3.31) is incorrect as stated. To obtain the correct version, we return to (5.3.30) and note that pole of $P(\tau)^{-1}$ at 0 is simple and the image of the residue, A , is \mathbb{C} (constant functions). Hence,

$$\check{u}_1(\tau) = \check{u}_0(\tau) + \check{u}_2(\tau),$$

$$\check{u}_0(\tau) := c_1 \int_{\mathbb{T}^n} \check{g}_1(\tau, x) dx / \tau, \quad \check{u}_2(\tau) := (P(\tau)^{-1} - A/\tau) \check{g}_1(\tau),$$

where $Q(\tau) := P(\tau)^{-1} - A/\tau$ satisfies the same estimates as $P(\tau)^{-1}$ but is holomorphic near 0. We note that $u_0 = c_0$ for $t > 0$.

We can now replace u_1 with u_2 as the constant term does not affect the energy estimate. Another way to look at this is changing the initial conditions from $u|_{t=0} = 0$, $\partial_t u|_{t=0} = f$, to $u|_{t=0} = -c_0$, $\partial_t u|_{t=0} = f$. This does not change energy $E(t)$.

In Step 3 we now have, for u_2 , which has the same energy of u_1 ,

$$\begin{aligned} \|e^{\beta t} u_2\|_{L^2(\mathbb{R}_+, H^1)} &= (2\pi)^{-\frac{1}{2}} \| \widehat{e^{\beta t} u_2} \|_{L^2(\mathbb{R}; H^1)} \\ &= (2\pi)^{-\frac{1}{2}} \| \check{u}_2(\cdot - i\beta) \|_{L^2(\mathbb{R}; H^1)} \\ &= (2\pi)^{-\frac{1}{2}} \| Q(\cdot - i\beta)^{-1} \check{g}_1(\cdot - i\beta) \|_{L^2(\mathbb{R}; H^1)} \\ &\leq C \| \check{g}_1(\cdot - i\beta) \|_{L^2(\mathbb{R}; L^2)}. \end{aligned}$$

The remainder of the proof is the same, with $u - c_0$ in place of u .

- page 129, Theorem 6.7: the statement about the constant h_0 should be made before (i) as $0 < h < h_0$ is also required for (ii) and (iii). (The statements are actually true for all values of h but that is not our concern.)
- page 130, line 5 of step 2: $K(-i, h)$ should be $K(i, h)$.
- page 135, line 3 from the bottom should read

$$\|b^w(x, hD)\| \leq \lambda + \frac{3\epsilon}{4},$$

for $0 < h < h(\epsilon)$. This follows, for instance from Theorem 4.30 applied to $(\lambda + \frac{3\epsilon}{4})^2 - (b^w(x, hD))^* b^w(x, hD)$ (see also Theorem 13.13).

- page 188, Remark: the second sentence should read “According to Theorem 8.10, if $a \in h^k S(m)$ for *some* $k \in \mathbb{R}$ and *some* order function m , then $T = a^w(x, hD)$ is tempered.
- page 190: (8.4.7) should be

$$\text{WF}_h(\exp(i\langle x, \omega \rangle / h^\alpha)) = \begin{cases} \mathbb{R}^n \times \{0\}, & \alpha < 1, \\ \mathbb{R}^n \times \{\omega\}, & \alpha = 1, \\ \emptyset & \alpha > 1. \end{cases}$$

that is, the division by h^α should be in the exponent.

- page 214, in formula (9.3.26), C is missing on the right; the expression should be:

$$\langle a^w(x, hD)u, u \rangle \geq -Ch \|u\|_{H_h^{\frac{m-1}{2}}}^2, \quad u \in \mathcal{S}.$$

- page 214, last line: $\text{spt} \tilde{\chi}_j$ should be $\text{spt} \tilde{\chi}_j$.
- page 276: the first two references to [H2] should be to [H3] and the next two to [H4].
- page 277, line 7: “canonical transformation” means “symplectic transformation”.
- page 278, Thm 12.4: The constant C in (12.2.3) depends on P and the neighbourhood containing $\text{WF}(u)$.
- page 279-280: In Theorem 12.5 and in its proof p (which determines the flow) needs to be replaced by p_0 . Since the proof reduces the general case to the normal form we apply the Jacobi Theorem 2.10 (the flow of ξ_1 becomes that of p_0) to make the conclusion about the general case.
- page 323: the proof in Step 2 is incorrect. First, replace (13.5.7) with

$$\|b^w(x, hD)\|^2 \geq \langle M_{|q|^2} T_\varphi u, T_\varphi u \rangle_{L_\Phi^2} - C_0 h, \quad \|u\|_{L^2} = 1.$$

and assume that $\Phi = |z|^2/2$. If the supremum of $|q|^2$ is achieved at, say, $z = 0$ then $\partial|q|^2(0) = 0$, and for $v = (2\pi h)^{-n/2}$

$$\begin{aligned} \langle M_{|q|^2} v, v \rangle_{L_\Phi^2} &= \frac{1}{(2\pi h)^n} \int_{\mathbb{C}^n} |q(z)|^2 e^{-|z|^2/h} dm(z) \\ &= \frac{1}{(2\pi h)^n} \int_{\mathbb{C}^n} (|q(0)|^2 + O(|z|^2)) e^{-|z|^2/h} dm(z) = |q(0)|^2 + \mathcal{O}(h). \end{aligned}$$

If the supremum is not attained then $|q(z_n)|^2 \rightarrow \sup |q|^2$, where $z_n \rightarrow \infty$. Since $|\partial^2 q| \leq C$, we have $\partial|q(z_n)|^2 \rightarrow 0$. We then choose n large enough so that $|q(z_n)|^2 \geq \sup |q|^2 - h$, and $|\partial q(z_n)| \leq h$, translate z_n to 0, and use the previous argument (or, directly, use $v(z) = (2/\pi h)^{n/2} (\det \Phi_{z\bar{z}})^{1/2} e^{\Psi(z, \bar{z})/h}$ as a test function).

- page 347, first displayed formula in §14.2.2: $S(\mathbb{R}^n)$ should be $S(\mathbb{R}^{2n})$.
- page 402: The estimate on $\|A\|^{2m}$ in the penultimate displayed formula should be JC^{2m} rather than JC^{2m+1} . As stated below the formula there are $2m - 1$ factors in the summands.