Assignment 7

1. (*Ahlfors, p.257, problem 1*) Prove directly that two circular annuli are conformally equivalent if and only if the ratio of their radii are equal.

2. (*Ahlfors, p.257, problem 2*) In the proof of theorem in the last lecture we constructed $g_j(z) = 2\partial_z \omega_j$ and then defined

$$\alpha_{kl} = i \int_{C_l} g_j(z) dz \,.$$

Show that $\alpha_{kl} = \alpha_{lk}$.

Hint: Integrate $\partial_z(\omega_k \omega_l)$ and $\partial_{\bar{z}}(\omega_k \omega_l)$ over the boundary using Green's formula written as follows

$$2 \int_R \partial_{\bar{z}} g(z) dx dy = -i \int_{\partial R} g(z) dz \,,$$

or, which is equivalent, follow the (corrected) hint in the text and use Theorem 19 on page 164.