Assignment 5

1. (Ahlfors, p.227, problem 1) Show that in any region \( \Omega \) the family of holomorphic functions with *positive real part* is normal. Under what added conditions is it locally bounded?
   **Hint:** Consider the functions \( \exp(-f(z)) \).

2. (Ahlfors, p.227, problem 3) If \( f(z) \) is analytic in the whole plane, show that the family formed by all functions \( f(kz) \), with \( k \) constant, is normal in the annulus \( r_1 < |z| < r_2 \), if and only if \( f \) is a polynomial.

3. (Ahlfors, p.227, problem 4) If the family of analytic (or meromorphic) functions is *not* normal in \( \Omega \), show that there exists a point \( z_0 \) such that \( F \) is not normal in any neighbourhood of \( z_0 \).
   **Hint:** Use a compactness argument.

4. Ahlfors, p.232, problem 1) If \( z_0 \) is real and \( \Omega \) is symmetric with respect to the real axis prove that the function mapping \( \Omega \) one-to-one and onto \( D(0,1) \), \( f(z_0) = 0 \), \( f'(z_0) \) real, satisfies
   \[
   f(\bar{z}) = \overline{f(z)}.
   \]