Assignment 5

1. (*Ahlfors, p.227, problem 1*) Show that in any region Ω the family of holomorphic functions with *positive real part* is normal. Under what added conditions is it locally bounded?

Hint: Consider the functions $\exp(-f(z))$.

2. (*Ahlfors, p.227, problem 3*) If $f(z)$ is analytic in the whole plane, show that the family formed by all functions $f(kz)$, with k constant, is normal in the annulus $r_1 < |z| < r_2$, if and only if f is a polynomial.

3. (*Ahlfors, p.227, problem 4*) If the family of analytic (or meromorphic) functions is *not* normal in Ω, show that there exists a point z_0 such that $\mathcal{F}$ is not normal in any neighbourhood of z_0.

Hint: Use a compactness argument.

4. *Ahlfors, p.232, problem 1*) If z_0 is real and Ω is symmetric with respect to the real axis prove that the function mapping Ω one-to-one and onto $D(0,1)$, $f(z_0) = 0$, $f'(z_0)$ real, satisfies

$$f(\bar{z}) = \overline{f(z)}\,.$$