Assignment 3

1. *(Ahlfors, p.186, problem 3)* The expression

\[ \{f, z\} = \frac{f'''(z)}{f'(z)} - 3 \left( \frac{f''(z)}{f'(z)} \right)^2, \]

is called the *Schwartzian derivative* of \( f \). If \( f \) has a multiple zero or pole find the leading term in the Laurent development of \( \{f, z\} \).

3. *(Ahlfors, p.190, problem 2)* Express

\[ \sum_{n=-\infty}^{\infty} \frac{1}{z^3 - n^3} \]

in closed form.

3. *(Ahlfors, p.193, problem 3)* Prove that for \(|z| < 1\)

\[ \prod_{k=0}^{\infty} (1 + z^{2^k}) = \frac{1}{1 - z}. \]

4. *(Ahlfors, p.193, problem 5)* Show that the function

\[ \theta(z) = \prod_{n=1}^{\infty} (1 + h^{2^{n-1}} e^z)(1 + h^{2^{n-1}} e^{-z}), \quad |h| < 1, \]

is analytic in the whole plane and satisfies the *functional equation*:

\[ \theta(z + 2 \log h) = h^{-1} e^{-z} \theta(z). \]