

Assignment 2

1. (*Ahlfors, p.96, problem 1*) Find a conformal mapping which maps the intersection of the discs $|z| < 1$ and $|z - 1| < 1$ onto $|z| < 1$. Choose the mapping so that symmetries are preserved.
2. (*Ahlfors, p.179, problem 4*) As a generalization of the Hurwitz theorem, prove that if $f_n \rightarrow f$ uniformly on compact subsets of Ω and f_n have *at most* m zeros in Ω then f is either identically zero, or has at most m zeros.
3. (*Ahlfors, p.179, problem 5*) Prove that

$$\sum_{n=1}^{\infty} \frac{nz^n}{1-z^n} = \sum_{n=1}^{\infty} \frac{z^n}{(1-z^n)^2}, \quad |z| < 1.$$

4. (*Ahlfors, p.184, problem 3*) Develop $\log(\sin z/z)$ into powers of z up to the term z^6 .