Assignment 1

1. (*Ahlfors, p.80, problem 4*) Show that any four distinct points can be carried by a linear transformation to positions $1, -1, k, -k$, where the value of k depends on the points. How many solutiosn are there, and how are they related?

2. (*Ahlfors, p.88, problem 2*) Suppose that

$$Sz = \frac{az+b}{cz+d}\,, \quad ad - cb = 1\,.$$

Show that S is

$$\begin{aligned} \text{elliptic} &\iff -2 < a+d < 2 \\ \text{parabolic} &\iff a+d = \pm 2 \\ \text{hyperbolic} &\iff 2 < a+d \text{ or } a+d < -2\,. \end{aligned}$$

3. (*Ahlfors, p.88, problem 3*) Show that any transformation which satisfies $S^n z = z$ for some integer n is necessarily elliptic.