Assignment 0

1. (*Ahlfors, p.28, problem 5*) Prove that the functions $f(z)$ and $\overline{f(\bar{z})}$ are simultaneously analytic.

2. (*Ahlfors, p.108, problem 4*) Compute $\int_{|z|=1} |z-1||dz|$. (Recall that $|dz|$ is the arc length element of integration: if $z = \gamma(t)$ then $|dz| = |\gamma'(t)|dt$.)

3. (*Ahlfors, p.108, problem 7*) If P is a polynomial and C is the circle $|z-a| = R$, what is the value of $\int_C P(z)d\bar{z}$? (The circle is oriented counter clockwise.)

4. (*Ahlfors, p.161, problem 5*) Show that if $f(z)$ is analytic and bounded for $|z| < 1$ and if $|\zeta| < 1$ then

$$f(\zeta) = \frac{1}{\pi} \int\int_{|z|<1} \frac{f(z)dxdy}{(1-\bar{z}\zeta)^2}.$$

This is known as *Bergman's kernel theorem.* To prove it, express the area integral in polar coordinates, thne transform the inside integral to a line integral which can be evaluated by residues. (There are also other ways to prove this famous formula.)