MATH 1B PRACTICE MIDTERM # 2

Problem 1. Determine the radii of convergence of

\[ a) \sum_{n=1}^{\infty} \frac{x^n}{n3^n} \]

\[ b) \sum_{n=1}^{\infty} \frac{x^{n-1}}{(n+1) \ln^2 n} \]

Do the series converge at the end points?

Problem 2. Find the power series expansions (centered at 0) of the following functions

\[ a) f(x) = x(4x^2 + 1)^{-1} \]

\[ b) f(x) = \sin 2x + x \cos 3x \]

Where are the expansions valid (i.e. for what values of \( x \) do they converge)?

Problem 3. Find the third order Taylor polynomial of \( 1/\sin x \) at \( \pi/2 \).

Problem 4. Write the integral

\[ \int_0^1 e^{x^3} \, dx \]

as an infinite series.

Problem 5. Consider the 5th order Taylor polynomial of \( \sin x \) at \( \pi \). For \( |x - \pi| \leq \pi \) determine which gives a more accurate measure of the quality of the approximation: the Taylor remainder estimate or the alternating series error bound.

Problem 6. Sketch the direction field of

\[ y' = \frac{x^2}{x^2 + 1} (y^2 - 1), \]

and determine the equilibrium solutions. Are they stable?

Problem 7. Solve the equation

\[ y' = 2 + 2y + x + xy. \]