1. From the first sentence, \(0.6k = 100\), so \(k = \frac{100}{0.6} N/m\). The system is thus modeled by the DE \(3y'' + \frac{100}{0.6} y = 0\). Its general solution is \(y(t) = c_1 \cos \frac{10}{3} t + c_2 \sin \frac{10}{3} t\). From our initial conditions \(y(0) = 0\), and \(y'(0) = 1.2\), we obtain that \(y = 0.36 \sin \frac{10}{3} t\).

2. Since \(k = \frac{24.3}{1.3} N/m\), about \(18.7 N/m\), if we let \(y\) represent the displacement from the equilibrium position, the system is modeled by the DE \(4y'' + 18.7y = 0\). Since \(\sqrt{\frac{18.7}{4}}\) is about 2.16, the general solution of this DE is \(y = c_1 \cos 2.16t + c_2 \sin 2.16t\). From our initial conditions, \(c_1 = -0.2\) and \(c_2 = 0\). Therefore \(y = -0.2 \cos 2.16t\).

3. Substituting \(k = 12N/m\), \(m = 2kg\), \(c = 14\) into equation 5, the system is modeled by the DE \(2y'' + 14y' + 12y = 0\). Its general solution (obtained via the methods of 17.1) is \(y = c_1 e^{-x} + c_2 e^{-6x}\). From our initial conditions, \(y(0) = 1\), \(y'(0) = 0\), we get \(c_1 = 1.2\), \(c_2 = -0.2\). Hence \(y = 1.2e^{-x} - 0.2e^{-6x}\).

9. We assume that our system can be modeled by the DE \(my'' + ky = F_0 \cos \omega_0 t\). The solution of the associated homogeneous equation \(my''_h + ky_h = 0\) has solution \(y_h = c_1 \cos \omega t + c_2 \sin \omega t\). If we use the method of undetermined coefficients and try \(y_p = A \cos \omega_0 t + B \sin \omega_0 t\), we obtain \(B = 0\), and

\[
A = \frac{F_0}{k - m\omega_0^2} = \frac{F_0}{m(k - \frac{k}{m})} = \frac{F_0}{m(\omega^2 - \omega_0^2)}
\]

Therefore \(y_p = \frac{F_0}{m(\omega^2 - \omega_0^2)} \cos \omega_0 t\), and the general solution is \(y = c_1 \cos \omega t + c_2 \sin \omega t + \frac{F_0}{m(\omega^2 - \omega_0^2)} \cos \omega_0 t\).

10. Now we assume that our system can be modeled by the DE \(my'' + ky = F_0 \cos \omega t\). We have the same solution to the associated homogeneous equation as above. Our usual attempt to find a solution to the inhomogeneous equation via the method of undetermined coefficients fails since \(A \cos \omega t + B \sin \omega t\) is precisely the characteristic solution. Therefore we try multiplying everything by \(t\), which works. If we try \(y_p = At \cos \omega t + Bt \sin \omega t\), we get \(A = 0\), and \(B = \frac{F_0}{2m\omega}\). Hence we may take \(y_p = \frac{F_0 t}{2m\omega} \sin \omega t\). Therefore the general solution to the inhomogeneous DE is \(y = c_1 \cos \omega t + c_2 \sin \omega t + \frac{F_0 t}{2m\omega} \sin \omega t\).

11. By assumption, \(\frac{p}{q} = \frac{p}{q}\) for integers \(p\) and \(q\). Therefore we may rewrite equation 6 as

\[
x(t) = c_1 \cos \frac{p\omega_0}{q} t + c_2 \sin \frac{p\omega_0}{q} t + \frac{F_0}{m(\omega^2 - \omega_0^2)} \cos \omega_0 t.
\]

Using the fact that \(\cos(t + 2\pi n) = \cos t\) and \(\sin(t + 2\pi n) = \sin t\) for every integer \(n\) and every real number \(t\),

\[
x(t + 2\pi n \frac{q}{\omega_0}) = x(t),
\]

for every integer \(n\) and every real number \(t\). Therefore the solution is periodic.

13. We will use the differential equation

\[LQ'' + RQ' + \frac{1}{C}Q = E(t)\]

Substituting the information we are given in the first sentence, this becomes

\[ Q'' + 2Q' + 500Q = 12. \]

The solution to the associated homogeneous equation (using the methods of 17.1) is \( Q_h = e^{-10t}(c_1 \cos 20t + c_2 \sin 20t) \). Either by inspection or by using the method of undetermined coefficients on a degree zero polynomial (i.e. a constant), we find the particular solution \( Q_p = 0.024 \). Now we have to determine coefficients of our general solution

\[
Q(t) = e^{-10t}(c_1 \cos 20t + c_2 \sin 20t) + 0.024 \\
I(t) = -10e^{-10t}(c_1 \cos 20t + c_2 \sin 20t) + e^{-10t}(-20c_1 \sin 20t + 20c_2 \cos 20t).
\]

From the initial conditions \( Q(0) = 0 \) and \( I(0) = 0 \), we obtain \( c_1 = -0.024 \) and \( c_2 = -.012 \). Finally,

\[
Q(t) = e^{-10t}(-0.024 \cos 20t + -.012 \sin 20t) + 0.024 \\
I(t) = 0.6e^{-10t} \sin 20t.
\]