§17.3

1. From the first sentence, $0.6k = 100$, so $k = \frac{100}{3}N/m$. The system is thus modeled by the DE $3y'' + \frac{100}{3}y = 0$. Its general solution is $y(t) = c_1 \cos\frac{10}{3}t + c_2 \sin\frac{10}{3}t$. From our initial conditions $y(0) = 0$, and $y'(0) = 1.2$, we obtain that $y = 0.36\sin\frac{10}{3}t$.

2. Since $k = \frac{24.3}{1.3}N/m$, about $18.7N/m$, if we let y represent the displacement from the equilibrium position, the system is modeled by the DE $4y'' + 18.7y = 0$. Since $\sqrt{\frac{18.7}{4}}$ is about 2.16, the general solution of this DE is $y = c_1 \cos 2.16t + c_2 \sin 2.16t$. From our initial conditions, $c_1 = -0.2$ and $c_2 = 0$. Therefore $y = -0.2\cos 2.16t$.

3. Substituting $k = 12N/m$, $m = 2kg$, $c = 14$ into equation 5, the system is modeled by the DE $2y'' + 14y' + 12y = 0$. Its general solution (obtained via the methods of 17.1) is $y = c_1e^{-x} + c_2e^{-6x}$. From our initial conditions, $y(0) = 1$, $y'(0) = 0$, we get $c_1 = 1.2$, $c_2 = -0.2$. Hence $y = 1.2e^{-x} - 0.2e^{-6x}$.

9. We assume that our system can be modeled by the DE $my'' + ky = F_0 \cos\omega_0 t$. The solution of the associated homogeneous equation $my_h'' + ky_h = 0$ has solution $y_h = c_1 \cos\omega t + c_2 \sin\omega t$. If we use the method of undetermined coefficients and try $y_p = A\cos\omega_0 t + B\sin\omega_0 t$, we obtain $B = 0$, and

$$A = \frac{F_0}{k - m\omega_0^2} = \frac{F_0}{m(\frac{k}{m} - \omega_0^2)} = \frac{F_0}{m(\omega^2 - \omega_0^2)}.$$

Therefore $y_p = \frac{F_0}{m(\omega^2-\omega_0^2)}\cos\omega_0 t$, and the general solution is $y = c_1\cos\omega t + c_2\sin\omega t + \frac{F_0}{m(\omega^2-\omega_0^2)}\cos\omega_0 t$.

10. Now we assume that our system can be modeled by the DE $my'' + ky = F_0\cos\omega t$. We have the same solution to the associated homogeneous equation as above. Our usual attempt to find a solution to the inhomogeneous equation via the method of undetermined coefficients fails since $A\cos\omega t + B\sin\omega t$ is precisely the characteristic solution. Therefore we try multiplying everything by t, which works. If we try $y_p = At\cos\omega t + Bt\sin\omega t$, we get $A = 0$, and $B = \frac{F_0}{2m\omega}$. Hence we may take $y_p = \frac{F_0 t}{2m\omega}\sin\omega t$. Therefore the general solution to the inhomogeneous DE is $y = c_1\cos\omega t + c_2\sin\omega t + \frac{F_0 t}{2m\omega}\sin\omega t$.

11. By assumption, $\frac{\omega}{\omega_0} = \frac{p}{q}$ for integers p and q. Therefore we may rewrite equation 6 as

$$x(t) = c_1\cos\frac{p\omega_0}{q}t + c_2\sin\frac{p\omega_0}{q}t + \frac{F_0}{m(\omega^2-\omega_0^2)}\cos\omega_0 t.$$

Using the fact that $\cos(t + 2\pi n) = \cos t$ and $\sin(t + 2\pi n) = \sin t$ for every integer n and every real number t,

$$x(t + 2\pi n\frac{q}{\omega_0}) = x(t),$$

for every integer n and every real number t. Therefore the solution is periodic.

13. We will use the differential equation

$$LQ'' + RQ' + \frac{1}{C}Q = E(t)$$

Substituting the information we are given in the first sentence, this becomes

$$Q'' + 2Q' + 500Q = 12.$$

The solution to the associated homogeneous equation (using the methods of 17.1) is $Q_h = e^{-10t}(c_1 \cos 20t + c_2 \sin 20t)$. Either by inspection or by using the method of undetermined coefficients on a degree zero polynomial (i.e. a constant), we find the particular solution $Q_p = 0.024$. Now we have to determine coefficients of our general solution

$$Q(t) = e^{-10t}(c_1 \cos 20t + c_2 \sin 20t) + 0.024$$

$$I(t) = -10e^{-10t}(c_1 \cos 20t + c_2 \sin 20t) + e^{-10t}(-20c_1 \sin 20t + 20c_2 \cos 20t).$$

From the initial condtions $Q(0) = 0$ and $I(0) = 0$, we obtain $c_1 = -0.024$ and $c_2 = -.012$. Finally,

$$Q(t) = e^{-10t}(-0.024 \cos 20t + -.012 \sin 20t) + 0.024 \qquad I(t) = 0.6e^{-10t} \sin 20t.$$