HW Solutions: 11.1, 11.2

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1 Section 11.1

Problem (4).

$$2/2, 3/5, 4/8, 5/11, 6/14, \ldots$$

Problem (6).

$$2, 8, 48, 384, 3840, \ldots$$

Problem (11).

$$a_n = 5n - 3$$

Problem (13).

$$a_n = (-2/3)^{n-1}$$

Problem (18).

$$a_n = \sqrt{n}/(1+\sqrt{n}) = 1/(\frac{1}{\sqrt{n}} + 1) \to 1/(0+1) = 1$$

Problem (22).

In absolute value, we have

$$\frac{|(-1)^n n^3|}{|n^3 + 2n^2 + 1|} = \frac{|n^3|}{|n^3 + 2n^2 + 1|}$$

which approaches 1, but the terms alternate in sign so the sequence does not converge.

Problem (26).

Converges to $\pi/2$ since $\lim_{x\to\infty} arctan x = \pi/2$.

Problem (49).

(a) $1060, 1123.60, 1191.02, 1262.48, 1338.23$
(b) diverges since $\lim_{n\to\infty}(1.06)^n = \infty$.

2 Section 11.2

Problem (2).

It means that $\lim_{n\to\infty} \sum_{i=1}^{n} a_i = 5$.

Problem (3).

see attached image

Problem (5).

see attached image

Problem (13).

This is a geometric series with $a = -2$ and $r = \frac{-5}{4}$; since $|r| > 1$, the series diverges.

Problem (15).

A geometric series with $r = 2/3$. $|r| < 1$, so it converges to

$$5\frac{1}{1-\frac{2}{3}} = 15$$

Problem (18).

A geometric series with $r = \frac{1}{\sqrt{2}}$, $|r| < 1$ so it converges to

$$\frac{1}{1-\frac{1}{\sqrt{2}}} = 2 + \sqrt{2}$$

Problem (27).

The series converges; it is the sum of the convergent series given by $a_n = 1/2$ and $b_n = 1/3$. n starts at 1 here so we have to shift the terms forward:

$$\sum_{n=1}^{\infty} \frac{3^n + 2^n}{6^n} = \sum_{n=1}^{\infty}(1/2)^n + \sum_{n=1}^{\infty}(1/3)^n = \frac{1}{2} \cdot \frac{1}{1 - 1/2} + \frac{1}{3} \cdot \frac{1}{1 - 1/3} = 1 + 1/2 = 3/2$$

Problem (32).

$0 < \cos(1) < 1$ so this is a convergent geometric series with sum

$$\frac{\cos 1}{1 - \cos 1}$$

Problem (34).

Diverges. To see why, suppose $\sum_{n=1}^{\infty}(\frac{3}{5^n} + \frac{2}{n})$ converges. then we can add another convergent series to it and combine the terms, and the result will still be convergent; add $\sum_{n=1}^{\infty} \frac{-3}{5^n}$ to get

$$\sum_{n=1}^{\infty}(\frac{3}{5^n} + \frac{2}{n}) - \sum_{n=1}^{\infty} \frac{3}{5^n} = \sum_{n=1}^{\infty}(\frac{3}{5^n} + \frac{2}{n} - \frac{3}{5^n}) = \sum_{n=1}^{\infty} \frac{2}{n}$$

which we know is not convergent since it is harmonic, a contradiction.

Problem (41).

We have a convergent geometric series with $|r| < 1$ if and only if x is such that $|x| < 3$

Problem (46).

$$a_n = \ln(1 + \frac{1}{n}) = \ln(\frac{n+1}{n})$$

As $n \to \infty$, $a_n \to \ln 1 = 0$. On the other hand,

$$\ln(\frac{n+1}{n}) = \ln(n+1) - \ln(n)$$

So this is a telescoping series; collapsing we get

$$s_n = \ln(n+1) - \ln(1) = \ln(n+1)$$

and $\lim_{n\to\infty} \ln(n+1) = \infty$ so the series diverges.

Problem (49).

A convergent series converges to the limit of its partial sums; here we are given the partial sums so can compute the limit directly

$$\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} s_n = \lim_{n\to\infty} \frac{n-1}{n+1} = 1$$

To find the a_n, we just observe that $a_1 = s_1 = 0$ and subtract to find the general term:

$$a_n = s_n - s_{n-1} = \frac{n-1}{n+1} - \frac{(n-1)-1}{(n-1)+1} = \frac{2}{n(n+1)}$$

Problem (50).

Letting $n = 1$, we find $a_1 = s_1 = 5/2$. For general n,

$$a_n = s_n - s_{n-1} = (3^{-}n2^{-n}) - [3 - (n-1)2^{-(n-1)}] = -\frac{n}{2^n} + \frac{n-1}{2^{n-1}} \cdot \frac{2}{2} = \frac{n-2}{2^n}$$

For the sum,

$$\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} s_n = \lim_{n\to\infty} 3 - n2^{-n} = 3$$

Since $\lim_{n\to\infty} n2^{-n} = 0$, which we can see by applying l'Hopistal's rule to the limit $\lim_{x\to\infty} \frac{x}{2^x}$ or by observing that exponential growth will overcome linear growth.