Homework for Mon 4/26
Chapter 17.2

13. The auxiliary equation of $y'' + 9y = 0$ is $r^2 + 9 = 0$ with roots $r = 3i, -3i$. Therefore neither e^{2x} nor $\sin(x)$ are solutions of the complementary homogeneous equation and we can use the trial function

$$y_p(x) = Ae^{2x} + (Bx^2 + Cx + D)\sin(x) + (Ex^2 + Fx + G)\cos(x)$$

14. The auxiliary equation of $y'' + 9y' = 0$ is $r^2 + 9r = 0$ with roots $r = 0, -9$. Therefore we can use the trial function

$$y_p(x) = (Ax + B)e^{-x}\cos(\pi x) + (Cx + D)e^{-x}\sin(\pi x)$$

15. The auxiliary equation of $y'' + 9y' = 0$ is $r^2 + 9r = 0$ with roots $r = 0, -9$. Then the constant functions are solutions of the complementary homogeneous equation and therefore we have to multiply the trial function corresponding to the constant 1 by x. Thus, the trial function should be

$$y_p(x) = Ax + (Bx + C)e^{9x}$$

17. The auxiliary equation of $y'' + 2y' + 10y = 0$ is $r^2 + 2r + 10 = 0$ with roots $r = -1+3i, -1-3i$ and thus the function $e^{-x}\cos(3x)$ is solution of the complementary homogeneous equation. Therefore we have to multiply the guess by x, i.e., the trial function should be

$$y_p(x) = x((Ax^2 + Bx + C)\cos(3x) + (Dx^2 + Ex + F)\sin(3x))e^{-x}$$

21. The auxiliary equation of $y'' - 2y' + y = 0$ is $r^2 - 2r + 1 = 0$ with root $r = 1$ (double root).
(a). The trial function should be $y_p(x) = Ae^{2x}$. Thus $y_p'(x) = 2Ae^{2x}$ and $y_p''(x) = 4Ae^{2x}$. Since $y_p(x)$ has to satisfy the equation $y_p'' - 2p_p' + y_p = e^{2x}$ we deduce that $(4A - 4A + A)e^{2x} = e^{2x}$ and so $A = 1$. Therefore the general solution of the equation is

$$y(x) = c_1e^x + c_2xe^x + e^{2x}$$

(b). Considering the guess $y_p(x) = u_1(x)y_1(x) + u_2(x)y_2(x)$ (where $y_1(x) = e^x$ and $y_2(x) = xe^x$) and imposing the condition $u_1'y_1 + u_2'y_2 = 0$ we obtain the equation

$$e^{2x} = u_1'y_1' + u_2'y_2' = u_1'e^x + u_2'e^x + u_2'xe^x = u_2'e^x$$

i.e. $u_2' = e^x$ so $u_2 = e^x$ and so $u_1'e^x = -xe^{2x}$. Thus, $u_1 = -xe^x + e^x$. Therefore the general solution is

$$y(x) = c_1e^x + c_2xe^x + (-xe^x + e^x)e^x + (e^x)xe^x = c_1e^x + c_2xe^x + e^{2x}$$

22. The auxiliary equation of $y'' - y' = 0$ is $r^2 - r = 0$ with roots $r = 0, 1$.
(a). Since e^x is solution of the homogeneous equation the trial function has to be $y_p(x) = Axe^x$. Thus $y_p'(x) = Ae^x + Axe^x$ and $y_p''(x) = 2Ae^x + Axe^x$. Plugging this into the equation we obtain $Ae^x = e^x$ and so $A = 1$. Therefore the general solution is

$$y(x) = c_1 + c_2 e^x + xe^x$$

(b). Considering the guess $y_p(x) = u_1(x)y_1(x) + u_2(x)y_2(x)$ (where $y_1(x) = 1$ and $y_2(x) = e^x$) and imposing the condition $u_1'y_1 + u_2'y_2 = 0$ we obtain the equation

$$e^x = u_1'y_1' + u_2'y_2' = u_2'e^x$$

Thus $u_2' = 1$ so $u_2 = x$ and the other equation implies $u_1' = -e^x$ so $u_1 = -e^x$ and the general solution is

$$y(x) = c_1 + c_2 e^x + (-e^x) + (x)e^x = c_1 + c_2' e^x + xe^x$$