§9.6

5. \( y' + 2y = 2e^x \)

Using \( P(x) = 2 \), we get the integrating factor

\[
I(x) = e^{\int 2 \, dx} = e^{2x}.
\]

Multiplying both sides by \( I(x) \), the DE becomes

\[
e^{2x} y' + 2e^{2x} y = (e^{2x} y)' = 2e^{3x}.
\]

Integrating both sides with respect to \( x \), we get

\[
e^{2x} y = 2e^{3x} + C,
\]

so

\[
y = 2e^x + Ce^{-2x}.
\]

8. \( x^2 y' + 2xy = \cos^2 x \)

The left-hand side is equal to \( (x^2 y)' \), so we integrate both sides (using the trig identity \( \cos^2 x = \frac{1}{2}(1 + \cos 2x) \)) to obtain

\[
x^2 y = \frac{x}{2} + \frac{1}{4} \sin(2x) + C,
\]

so

\[
y = \frac{1}{2x} + \frac{1}{4x^2} \sin(2x) + \frac{C}{x^2}.
\]

15. \( y' = x + y, \ y(0) = 2 \)

The DE \( y' - y = x \) is linear with \( P(x) = -1 \). Therefore we take as our integrating factor \( I(x) = e^{-x} \). Multiplying both sides by \( I \), we obtain

\[
e^{-x} y' - e^{-x} y = (ye^{-x})' = xe^{-x}.
\]

Integrating both sides with respect to \( x \) (use integration by parts on the RHS with \( u = x \) and \( dv = e^{-x} \)) to obtain

\[
y e^{-x} = -xe^{-x} - \int xe^{-x} \, dx = -xe^{-x} - e^{-x} + C.
\]

Multiplying both sides by \( e^x \),

\[
y = -x - 1 + Ce^x.
\]

Using our initial condition, \( y(0) = -1 + C = 2 \), so \( C = 3 \). Our solution to the IVP is

\[
y = -x - 1 + 3e^x.
\]
17. 

\[ \frac{dv}{dt} - 2tv = 3t^2 e^{t^2}, \quad v(0) = 5. \]

This is a linear DE with \( P(t) = -2t \). Therefore we use the integrating factor 

\[ I(t) = e^{\int -2t \, dt} = e^{-t^2}. \]

Multiplying both sides by \( I(t) \),

\[ e^{-t^2} \frac{dv}{dt} - 2te^{-t^2} v = (e^{-t^2} v)' = 3t^2. \]

Integrating both sides with respect to \( x \),

\[ e^{-t^2} v = t^3 + C. \]

Applying our initial condition, \( v(0) = C = 5 \). Therefore

\[ e^{-t^2} v = t^3 + 5, \]

and multiplying both sides by \( e^{t^2} \),

\[ v = t^3 e^{t^2} + 5e^{t^2}. \]

29. Substituting the given values into the DE gives

\[ \frac{dQ}{dt} + 4Q = 12. \]

This is a linear DE with \( P(t) = 4 \). We use the integrating factor \( I(x) = e^{4t} \), multiply both sides by the integrating factor and get

\[ (e^{4t} Q)' = 12e^{4t}, \]

so \( e^{4t} Q = 3e^{4t} + C \). Our initial condition, \( Q(0) = 3 + C = 0 \) tells us that \( C = -3 \), so substituting this value for \( C \) and multiplying both sides by \( e^{-4t} \) gives

\[ Q = 3 - 3e^{-4t}. \]