§9.6

5. $y' + 2y = 2e^x$

Using $P(x) = 2$, we get the integrating factor

$$I(x) = e^{\int 2dx} = e^{2x}.$$

Multiplying both sides by $I(x)$, the DE becomes

$$e^{2x}y' + 2e^{2x}y = (e^{2x}y)' = 2e^{3x}.$$

Integrating both sides with respect to x, we get

$$e^{2x}y = 2e^{3x} + C,$$

so

$$y = 2e^x + Ce^{-2x}.$$

8. $x^2y' + 2xy = \cos^2 x$

The left-hand side is equal to $(x^2y)'$, so we integrate both sides (using the trig identity $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$ to obtain

$$x^2y = \frac{x}{2} + \frac{1}{4}\sin(2x) + C,$$

so

$$y = \frac{1}{2x} + \frac{1}{4x^2}\sin(2x) + \frac{C}{x^2}.$$

15. $y' = x + y$, $y(0) = 2$

The DE $y' - y = x$ is linear with $P(x) = -1$. Therefore we takes as our integrating factor $I(x) = e^{-x}$. Multiplying both sides by I, we obtain

$$e^{-x}y' - e^{-x}y = (ye^{-x})' = xe^{-x}.$$

Integrating both sides with respect to x (use integration by parts on the RHS with $u = x$ and $dv = e^{-x}$) to obtain

$$ye^{-x} = -xe^{-x} + \int e^{-x}dx = -xe^{-x} - e^{-x} + C.$$

Multiplying both sides by e^x,

$$y = -x - 1 + Ce^x.$$

Using our initial condition, $y(0) = -1 + C = 2$, so $C = 3$. Our solution to the IVP is

$$y = -x - 1 + 3e^x.$$

17.
$$\frac{dv}{dt} - 2tv = 3t^2 e^{t^2}, \qquad v(0) = 5.$$

This is a linear DE with $P(t) = -2t$. Therefore we use the integrating factor
$$I(t) = e^{\int -2t dt} = e^{-t^2}.$$

Multiplying both sides by $I(t)$,
$$e^{-t^2}\frac{dv}{dt} - 2te^{-t^2}v = (e^{-t^2}v)' = 3t^2.$$

Integrating both sides with respect to x,
$$e^{-t^2}v = t^3 + C.$$

Applying our initial condition, $v(0) = C = 5$. Therefore
$$e^{-t^2}v = t^3 + 5,$$

and multiplying both sides by e^{t^2},
$$v = t^3e^{t^2} + 5e^{t^2}.$$

29. Substituting the given values into the DE gives
$$\frac{dQ}{dt} + 4Q = 12.$$

This is a linear DE with $P(t) = 4$. We use the integrating factor $I(x) = e^{4t}$, multiply both sides by the integrating factor and get
$$(e^{4t}Q)' = 12e^{4t},$$

so $e^{4t}Q = 3e^{4t} + C$. Our initial condition, $Q(0) = 3 + C = 0$ tells us that $C = -3$, so substituting this value for C and multiplying both sides by e^{-4t} gives $Q = 3 - 3e^{-4t}$.