§9.3

$$1. \qquad \frac{dy}{dx} = \frac{y}{x}$$

We write the differential equation as

$$\frac{dy}{y} = \frac{dx}{x}$$

and integrate both sides to obtain

$$\ln y = \ln x + C.$$

Taking the exponential of both sides,

$$y = e^C x,$$

or equivalently $y = Kx$.

$$3. \qquad (x^2+1)y' = xy$$

We write the differential equation as

$$\frac{dy}{y} = \frac{xdx}{1+x^2},$$

integrate...

$$\ln y = \frac{1}{2}\ln(1+x^2) + C = \ln(\sqrt{1+x^2}) + C$$

apply exp to both sides and absorb the constant...

$$y = A\sqrt{1+x^2}.$$

$$6. \qquad \frac{du}{dr} = \frac{1+\sqrt{r}}{1+\sqrt{u}}$$

Write

$$(1+\sqrt{u})du = (1+\sqrt{r})dr;$$

integrate to get

$$u + \frac{2}{3}u^{\frac{3}{2}} = r + \frac{2}{3}r^{\frac{3}{2}} + C.$$

And leave this as our solution.

$$9. \qquad \frac{du}{dt} = 2 + 2u + t + tu$$

We have $\frac{du}{dt} = (2+t)(1+u)$, which we rewrite as

$$\frac{du}{1+u} = (2+t)dt,$$

integrate...

$$ln(1+u) = 2t + \frac{t^2}{2} + C$$

apply exp, absorb the constant, and subtract 1 from both sides

$$u = Ae^{2t+\frac{t^2}{2}} - 1.$$

$$11. \qquad \frac{dy}{dx} = y^2 + 1, \qquad y(1) = 0$$

We rewrite the DE

$$\frac{dy}{y^2+1} = dx$$

integrate...

$$\tan^{-1} y = x + C$$

evaluate at $x = 1$,

$$\tan^{-1} y(1) = \tan^{-1} 0 = 0 = 1 + C,$$

so $C = -1$. Therefore $y = \tan(x-1)$.

$$13. \qquad x\cos x = (2y + e^{3y})y', \qquad y(0) = 0$$

We separate,

$$x \cos x dx = (2y + e^{3y})dy,$$

integrate (integrate the left side by parts with $u = x$, $dv = \cos x dx$)

$$-x\sin x - \cos x = y^2 + \frac{e^{3y}}{3} + C,$$

evaluate at $(x, y) = (0, 0)$

$$-1 = \frac{1}{3} + C,$$

so that $C = -\frac{4}{3}$.

31. Solve the initial-value problem in Exercise 27 in Section 9.2 to find an expression for the charge at time t. Find the limiting value of the charge.

We have the differential equation

$$5\frac{dQ}{dt} + \frac{1}{.05}Q = 60.$$

Rewriting this equation,

$$\frac{dQ}{dt} = 12 - 4Q$$

rewrite...

$$\frac{dQ}{12 - 4Q} = dt$$

integrate...

$$-.25\ln(12 - 4Q) = t + C,$$

so

$$\ln(12 - 4Q) = -4t + B.$$

Evaluating at (Q,t) = (0,0) gives

$$B = \ln 12.$$

Expifying both sides,

$$Q = -3e^{-4t} + 3.$$

As $t \to \infty$, this tends to 3.