Math 1B, Prof Zworski
Section 17.1

1: $y'' - 6y' + 8y = 0$. The auxiliary equation is $r^2 - 6r + 8 = 0$. The roots are $r = 2, 4$. Hence the general solution is

$$y(x) = c_1 e^{2x} + c_2 e^{4x}$$

2: $y'' - 4y' + 8y = 0$. The auxiliary equation is $r^2 - 4r + 8 = 0$. The roots are $r = (4 \pm \sqrt{-16})/2 = 2 \pm 2i$. Hence the general solution is

$$y(x) = e^{2x}(c_1 \cos 2x + c_2 \sin 2x)$$

6: $3y'' - 5y' = 0$. The auxiliary equation is $3r^2 - 5r = 0$. The roots are $r = 0, 5/3$. Hence the general solution is

$$y(x) = c_1 + c_2 e^{5x/3}$$

9: $4y'' + y' = 0$. The auxiliary equation is $4r^2 + r = 0$. The roots are $r = 0, -4$. Hence the general solution is

$$y(x) = c_1 + c_2 e^{-4x}$$

17: $2y'' + 5y' + 3y = 0$. The auxiliary equation is $2r^2 + 5r + 3 = 0$. The roots are $r = -1, -3/2$. Hence the general solution is

$$y(x) = c_1 e^{-x} + c_2 e^{-3x/2}$$

So $y'(x) = -c_1 e^{-x} - (3/2)c_2 e^{-3x/2}$. Plugging in the initial conditions $y(0) = 3, y'(0) = -4$, we get the equations

$$c_1 + c_2 = 3 \qquad ; \qquad -c_1 - (3/2)c_2 = -4$$

The solution is $c_1 = 1$, $c_2 = 2$. The solution to our initial value problem is $y(x) = e^{-x} + 2e^{-3x/2}$.

19: $4y'' - 4y' + y = 0$. The auxiliary equation is $4r^2 - 4r + 1 = 0$. The roots are $r = 1/2, 1/2$. Hence the general solution is

$$y(x) = c_1 e^{x/2} + c_2 x\, e^{x/2}$$

So $y'(x) = c_1 e^{x/2} + c_2 e^{x/2} + (c_2/2)x\, e^{x/2}$. Plugging in the initial conditions $y(0) = 1, y'(0) = -1.5$, we get the equations

$$c_1 = 1 \qquad ; \qquad c_1 + c_2 = -1.5$$

The solution is $c_1 = 1$, $c_2 = -2.5$. The solution to our initial value problem is thus $y(x) = e^{x/2} - 2.5x\, e^{x/2}$.