2018 PUMaC Math Bowl
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Round 1

1. Intending to hole himself up for months working on the symmetries of the Leech lattice, this mathematician accidentally solved it on the same Saturday on which he began. He first described the Look-and-Say Sequence, and with Richard Guy and Elwyn Berlekamp, he coauthored *Winning Ways for Your Mathematical Plays*. For the point, name this emeritus professor of mathematics at Princeton who invented a namesake Game of Life.
Answer: John Horton **Conway**

2. This website's blog is named the "Blag" and one entry includes an intercepted "imaginary friends" pun, while another introduces the "Very Weak" and "Extremely Weak" Goldbach conjectures. Mathematicians coauthor a paper with zombie Paul Erdos [air-DISH] and taunt Physicists and Sociologists on a line in two entries to this website that also features the section "What If?" Updated three times a week is, for the point, what Randall Munroe-drawn STEM-based webcomic?
Answer: **XKCD**.com

3. Members of this family of functions express the displacement of a particle moving in 1D with constant jerk as a function of time. While there is a simple formula for the zeroes of any these functions, discovered in 16th century in Italy, the formula can involve square roots of negative numbers even when all the zeros are ultimately real. In fact, it was in the course of studying these functions that imaginary numbers were first discovered. For the point, name this family of functions that includes the lowest degree polynomials that have critical points but no local maxima or minima.
Answer: **cubic polynomials**

4. To prove the Urysohn lemma, one of these is constructed for each rational number, and the axioms T1 through T6 ensure that disjoint ones may accomplish certain feats. A topological space is a set along with a declaration of these objects, and it is compact if any infinite cover using these mathematical objects has a finite subcover. For the point, the interval (0,1) not containing its endpoints is an example in the real line of what, in contrast to those that are closed?
Answer: **open set**, prompt on "open", don't require "set" on last line, prompt on "open interval"

5. This property is weaker than commutativity but stronger than being a Euclidean domain, and is held by rational linear combinations of the 22nd roots of unity but not the 23rd. This property also ceases to hold in the set of integer combinations of the square root of negative five. The fundamental theorem of arithmetic states, for the point, that the integers have what property, which allows each number to be written as the product of primes in exactly one way?
Answer: **unique factorization** (accept **UFD** before "FTP")

6. James Madison was the first person with this role, although now there are close to 3,000 such people. Mathematicians to have held this designation include Charles Fefferman and Terence Tao. These people may live in

facilities surrounding Cleveland Tower, and in mathematics have funding for five years. For the point, name this group of students who study in New Jersey and who pursue PhDs.
Answer: **graduate student at Princeton** or equivalents (be generous but don't accept anything like "undergrad at Princeton")

7. This person studied under Alonzo Church and conames a "thesis" with him, and in 1936 introduced the notion of a "Universal Machine". Later, building on the Polish development of the *bombe*, this person lead a codebreaking team at Bletchley Park. For the point, name this pioneer of computer science who cracked the Enigma device, the subject of the film *The Imitation Game*.
Answer: Alan **Turing**

8. This was once known as the Staircase of Mount Meru, though it gained its present name in the 18th century, and earlier Omar Khayyam used it to compute n-th roots. The Sierpinski gasket arises from selecting certain moduli, and it is subject to the hockey stick identity. For the point, name this configuration of integers where the k-th element in the n-th row is equal to n choose k.
Answer: **Pascal's triangle**

9. This mathematician proved the ergodicity of earthquake flow in Teichmuller space, and more broadly studied dynamics of Riemann surfaces and moduli spaces, which has been described as billiards on constantly mutating tables. This professor at Princeton from 2004 to 2009 was the first Iranian to receive mathematics' highest prize. For the point, name this 2014 Fields Medalist who passed away in 2017.
Answer: Maryam **Mirzakhani**

10. This 1982 MacArthur Fellow worked for George McGovern's 1972 presidential campaign and briefly pursued graduate degrees in economics at the University of Wisconsin-Madison and applied mathematics at Princeton. This founder of M-theory joined the Institute for Advanced Study in 1981, and coined the name "topological quantum field theory." For the point, name the only physicist to win a Fields Medal.
Answer: Edward **Witten**

11. The dope vector can give information about the memory location of this data type. The F# [F sharp] programming language allows for parallelized computing over these data structures, and in C languages they usually all contain elements of the same data type. For the point, name this iterable data structure with elements that can often be accessed with square brackets following the object name.
Answer: **array** or **list**

12. The Law of Large Numbers ensures a phenomenon by this name occurs for a random variable with probability 1. It has unconditional, pointwise and uniform varieties. In a complete space, this property is equivalent to Cauchiness. For the point, name this term for when a sequence of points gets arbitrarily close to a unique value.
Answer: **convergence** (or word forms)

13. *The Grand Design* and *The Universe in a Nutshell* are two nonfiction works by this scientist. This frequent collaborator with Gary Gibbons also worked with Roger Penrose on gravitational singularities in general relativity. This namesake of a classification of radiation also write *A Brief History of Time*. For the point, name this British cosmologist who was the subject of 2014's *The Theory of Everything*.
Answer: Stephen **Hawking**

14. This operation has sinusoidal eigenfunctions, and can be generalized through the Hessian matrix. This operation can be performed on elements of $C^2(R)$, its result for a quadratic is constant, and it names a "test" for determining the convexity of a function's graph. For the point, name this operation which can be done by taking d^2/dx^2 and is denoted f'' [f double prime].
Answer: **second derivative**, do not accept or prompt on first derivative

15. This is the subject of the fifth Zermelo-Fraenkel axiom. This operation may be done infinitely-many times to open sets in a topological space to obtain another open set. It corresponds with the logical operation "or", and contrasts with the intersection operation. For the point, name this operation which returns the elements of both of two sets.
Answer: set **union**

TIEBREAKER: What is the product of the number of Princeton mathematics majors this year, the number of graduate students, and the number of full-time faculty and staff?
Answer: 73 * 72 * 105 = 551880

Round 2

1. An intermediate step in addition on elliptic curves is the construction of a point with this relation to the other two. The orthocenter, circumcenter, and centroid of any triangle have this property. Moreover, there is a unique circle passing through any set of three points without this property, and this property is the only thing that can prevent three points from defining a triangle. For the point, name this property held by three points all on the same line.
Answer: **collinear**ity

2. This process is an ingredient in a wide variety of thought experiments, including the Sleeping Beauty Problem and the St. Petersburg Paradox, and it can also be used to model Martin Gardner's Boy or Girl Paradox. This process constitutes a Bernoulli trial with p = ½, and although one must have money to perform it, it is extremely cheap. For the point, name this probabilistic process that is perhaps a common method of settling disagreements.
Answer: **coin tossing**

3. Project Matterhorn was an early attempt at studying this process. Currently, scientists use stellarators and tokamaks to better understand it, and one leader in attempts to replicate this process is the Princeton Plasma Physics Laboratory. For the point, name this physical process involving the combination of nuclei, and which is believed to be a clean source of power for the future.
Answer: nuclear **fusion**

4. One person with this position was a minister and signatory of the Articles of Confederation and the Declaration of Independence, and appears in John Trumbull's painting of its signing, and later oversaw the acquisition of the Rittenhouse orrery; that person was John Witherspoon. Another person in this role, Shirley Tilghman, oversaw the implementation of grade deflation, but that was overturned in 2014. For the point, name this highest office, currently held by Christopher Eisgruber, at the school where we presently sit, who is elected by a Board of Trustees.
Answer: **President** of **Princeton** University

5. This constant is the coefficient of the leading term of the differential equation of the p ["pay"] function. To prove Goursat's Theorem, a region is recursively partitioned into this many subregions. Theorems attributed to Jacobi and Lagrange give information about writing positive integers as the sum of this many squares. For the point, name this constant that appears in complex analysis as the number of sides in a parallelogram.
Answer: **four**

6. The original location of this building is now occupied by an East Asian Studies library and is connected to Frist Campus Center; that is now Jones Hall. This location's namesake was a President of the AMS. It has underground connections to Jadwin Hall and Lewis Library. For the point, name this home of the Princeton Mathematics Department.
Answer: **Fine** Hall

7. This issuer of checks from the Bank of San Serriffe invented the serifed Computer Modern, and with Vaughan ["von"] Pratt invented a string-searching algorithm. Leslie Lamport augmented a project of this individual's, lending the prefix "La" ["lah"], and if you find a mistake in one of his writings he will give you $2.56. For the point, name this author of *The Art of Computer Programming*.
Answer: Donald **Knuth**

8. After sampling only 30 values, this mathematician correctly conjectured a tight bound of 11/2 in an exponent regarding the tau function. In 1976, other writings were discovered in this mathematician's "lost notebook." At Cambridge, he studied under Hardy and Littlewood. This mathematician's constant is also known as the taxicab number, and equals 1729. For the point, name this self-taught Indian mathematician of the early 20th century.
Answer: Srinivasa **Ramanujan**

9. This work's fourth volume on geometry was planned but ultimately nixed. It was an attempt to circumvent paradoxical sets allowed by Gottlob Frege's ["fray-guz"] model, but it was superseded by Godel's ["GUR-dels"] incompleteness theorem. In Theorem 54.43, after approximately 340 pages, this text proves that 1+1=2. For the point, name this text in foundations of mathematics by Russell and Whitehead.
Answer: ***Principia*** *Mathematica*

10. Cocks, Ellis and Williamson are three unheralded pioneers in this field. Concepts in this field include meet-in-the-middle and zero-knowledge. In this field, protocols include CCA. The discrete logarithm problem is why Diffie-Hellman works, and another example includes its method of RSA. For the point, name this area concerned with encryption and decryption.
Answer: **cryptography**, prompt on "codebreaking"

11. This statement is equivalent to an infinite set having the same cardinality as its cartesian square. It's not the continuum hypothesis, but Godel ["GUR-del"] and Cohen showed that it is independent from Zermelo-Fraenkel set theory. This statement implies that it is possible to partition a sphere into two identical spheres, and is known as the Banach-Tarski paradox. For the point, name this controversial statement in set theory which asserts that from an uncountable collection, one may select a representative.
Answer: **Axiom of Choice** (**AC**), accept **Zorn's Lemma** (**ZL**) and the **Well-Ordering Principle** (**WOP**)

12. Nicholas Katz was once the sole student in a class taught by this professor, and Goro Shimura once reacted to this person's news with "I told you so." That news was a proof of the modularity theorem. To complete that result, also known as the Taniyama-Shimura-Weil conjecture, this mathematician collaborated with former student Richard Taylor. For the point, name this mathematician who proved Fermat's Last Theorem.
Answer: **Andrew Wiles**

13. George Stoney coined this term, six years before its existence was confirmed. Millikan and Fletcher computed its charge using the oil-drop experiment, and its discoverer, J.J. Thomson, initially argued for its part in the plum-pudding model, before knowing that the nucleus existed. For the point, name this component of the atom which is lighter than the neutron and proton.
Answer: **electron**

14. One of these arguments is known as "affirming the consequent," and another is known as "denying a conjunct." One can be used to show that all triangles are isosceles, and another common mathematical one involves division by zero. In economics, an oft-cited one is that of sunk costs, and in debate they can include "straw man" and "appeal to authority." For the point, name this type of faulty philosophical, logical or mathematical argument.
Answer: **fallacy**

15. In category theory, this concept has direct and inverse variants. It is defined on a Banach space such that for all sequences, it is linear, positive, and shift invariant. It has one definition, formalized by Cauchy and Weierstrass, which typically uses either the variables epsilon, little n, and big N, or the variables epsilon and delta. It is used in calculus to define continuity and derivatives; L'Hopital's rule and the Squeeze theorem use these. It is possible for these values to not exist or be positive or negative infinity. For the point, name the value that a function approaches.
Answer: **limit**

TIEBREAKER: What is the product of the number of course offerings by the Princeton Math Department for Fall 2018 and the number of spots available in graduate courses this semester?
Answer: 33 * 294 = 9702

Round 3

1. Emil Artin showed that a field acted on by a group of functions with fixed field F, taken modulo F, is this type of extension. A correspondence named with this adjective exists between intermediate fields and the group of automorphisms of those fields. For the point, give this type of theory which was used by Niels Abel to prove the nonexistence of the quintic formula, and which was named for a 19th century French mathematician.
Answer: **Galois**

2. Along with Philippe Michel, this mathematician used ergodic methods to generalize subconvexity results for L-functions, and more broadly he is renowned for using varied techniques to resolve questions on arithmetic equidistribution. In 2018, this number theorist transferred from Stanford University to the Institute for Advanced Study, and is the only Australian to have medaled at the IMO and the IPhO ["eye-FO"]. For ten points, name this 2018 Fields Medalist.
Answer: Akshay **Venkatesh**

3. An Azumaya algebra over this structure is an algebra that is free and of finite rank greater than or equal to 1. This structure's concept initially arose when Dedekind attempted to prove Fermat's Last Theorem, introducing the terms "ideal" and "module." Familiar examples of these structures include the integers and the integers modulo N. For the point, name this structure consisting of a set with two binary operations, which is an abelian group under addition.
Answer: **ring**

4. Dick Kazmaier is the most-acclaimed participant in this group's history. It lost the first-ever match of its activity in 1869, but since then has won more national championships than any other. This group plays at Powers Field. Currently coached by Bob Surace is, for the point, what Ivy League gridiron team?
Answer: **Princeton** University men's **football** team

5. Grover names a specialized approach for accomplishing this task, and an efficient classical method for accomplishing it uses heuristics to improve its speed; that is known as A* [A star]. Depending on the structure, the depth-first or breadth-first variant may be preferable. Naively, it takes expected linear time, or it can be done through a "binary" approach. For the point, name this computational task that involves finding a given member of a collection.
Answer: **search**ing algorithms

6. This mathematician's first wife was named Eleanor Stier. An embedding theorem named for this mathematician asserts the existence of an isometry from any Riemannian manifold into a real space. This mathematician did not receive the 1956 Fields Medal possibly because Ennio de Giorgi simultaneously affirmed Hilbert's 19th problem. Once known as the "Ghost of Fine Hall," beginning in the 1950s, this mathematician suffered from schizophrenia, and in 1994, this mathematician received a Nobel Prize in Economics. For the point, name this mathematician immortalized in Sylvia Nasar's book *A Beautiful Mind*.
Answer: John **Nash**

7. This scientist was urged to pursue mathematics in order to maximize distance with the field of study of Lord Byron. Later, this scientist noted the use of punch cards in mechanized looms as a possible route of scientific advancement.

This scientist's Note G provides a method for computing Bernoulli numbers, and was intended to operate on Charles Babbage's analytical engine. For the point, name this first computer programmer.
Answer: (Augusta) Ada **Lovelace** / **King**, prompt on "Byron" and require first name

8. The variant of this statement for finite fields won Pierre Deligne ["duh-LEEN"] a Fields Medal in 1974, and it generalizes using Dirichlet characters and L-functions. David Hilbert said that he would like to know the veracity of this statement more than any other, and in 2018, Michael Atiyah presented a false proof of it. For the point, name this conjecture about the placement in the complex plane of zeros of the zeta function, a fundamental open problem in number theory.
Answer: **Riemann Hypothesis**

9. The world's largest computing grid is affiliated with this place, and locations here include ATLAS, TOTEM, MoEDAL ["modal"], and ALICE. This past summer, improvements planned to complete by 2026 began that will increase its luminosity by an order of magnitude. This facility faced a major setback after a 2008 magnet quench, and has been subject to conspiracy theories that it will cause tiny black holes to appear. In 2012, the Higgs Boson was discovered at this European facility. For the point, name this particle accelerator located underground near Geneva, and operated by CERN ["sern"].
Answer: **L**arge **H**adron **C**ollider

10. Paul Seymour was one of two mathematicians who proved the n=6 case of the Hadwiger conjecture, which is about these mathematical objects. Seymour and Maria Chudnovsky received the Fulkerson Prize for a "strong" result on the "perfect" type of these mathematical objects, and Chudnovsky devised a polynomial-time algorithm for recognizing perfect ones. Seymour simplified Appel and Haken's proof of the four colour theorem, which states that any planar of these can have a four-color assignment. For the point, name this mathematical object consisting of vertices and edges.
Answer: **graph**, do not accept or prompt on anything more specific

11. This function appears under the radical in the expression for spherical harmonics, and the term sqrt(2πn) [square root of two pi n] is included in Stirling's approximation of this function. This function is the coefficient for the nth derivative of x^n [x to the n], and is in the denominator of each coefficient in the Taylor series for e^x [e to the x]. For the point, give this function which is the product of the first n positive integers.
Answer: **factorial** or **gamma** function

12. This result, which is known to some as the Gougu theorem, can be generalized to inner product spaces, giving the parallelogram law, and another generalization involving parallelograms is known as Pappus' area theorem. One proof of this result, provided by Euclid, begins with a square subdivided into two rectangles. For the point, name this theorem that relates the three sides of a right triangle.
Answer: **Pythagorean theorem**

13. According to folklore, at one of these events in the early 2000s began the tradition of calling its leaders "czar", and the largest of them can attract participants from more than six countries. USMCA is a newly-formed consortium of these, and it includes CHMMC, CMIMC, and PUMaC. For the point, at what events do hundreds of teenagers---like you---arrive at a campus---like this one---to complete an examination---like the the one you just took?

Answer: **college-run math contests** for high schoolers, prompt on "math contests" or "high school math contests"; "contests" can be replaced by "competitions" or "tournaments" or other synonyms

14. Markov's inequality uses *this* to give an upper bound on the probability that a function of a random variable is greater than or equal to a constant. It is also used in Jensen's, Holder's, and Minkowski's inequalities. Properties of this value include non-degeneracy, monotonicity and linearity. For the point, name the value which represents the long-run average of an experiment's result.
Answer: **expected value** or **expectation**

15. In higher dimensions, this result generalizes to a product of matrices, and it can incorrectly be "proved" by cancelling terms in the multiplication of non-fractional quotients. Inexplicably, this result was not used in any published text of Euler, even though it was known at his time. This formula is used in integration by substitution and the quotient rule, and it was first used by Leibniz to calculate the derivative of $sqrt(a+bz+cz^2)$. For the point, name this formula which finds the derivative of the composition of two functions as $f'(g(x))g'(x)$ [f prime g of x times g prime of x].
Answer: **chain rule**

TIEBREAKER: What is the product of the number of students currently pursuing a certificate in Applied and Computational Mathematics and the number of full-time faculty at Princeton?
Answer: 1261 * 26 = 32786

Round 4

1. This mathematician simplified Conway's proof of the 15 Theorem, and with Jon Hanke proved the 290 Theorem. Both of these results are part of his work in number theory, which also includes extended Gauss' work on binary quadratic forms. For the point, name this professor of mathematics at Princeton who was among the 2014 Fields Medalists.
Answer: Manjul **Bhargava**

2. Upon seeing the work of one person with this surname, Gauss told another person with this surname that he had made those discoveries years earlier but deemed them too dangerous to publish. The younger person with this surname remarked, "Out of nothing I have created a strange new universe;" his father, Farkas, had spent much of his adult life attempting to prove the parallel postulate. For the point, what was the surname of the Hungarian mathematician who, along with Lobachevsky, established non-Euclidean geometry?
Answer: (Janos/Johann or Farkas) **Bolyai**

3. This result was known as the aureum theorema to the first person to prove it. The most commonly taught proof of this theorem was discovered by Eisenstein, a student of Gauss, who at one point struggled to prove it but eventually proved it in eight different ways in his career. For the point, name this result relating to solutions of modular quadratic diophantine equations, often stated involving Legendre symbols of prime numbers.
Answer: **Quadratic Reciprocity**, prompt on **QR**

4. When a function independent of the azimuth is integrated in polar coordinates, this value can be brought outside the integral as a constant factor. Although it is not found in the standard formula for the area of a circle, it is found in the standard formulas for the probability density function of the normal distribution, the Fourier transform, and the circumference of a circle. Each year on June 28, it is celebrated by people who consider it to be a more fundamental constant than pi. Name this value that is the number of radians in one full turn.
Answer: **2pi** or **tau**

5. This mathematician inadvertently used an incorrect theorem of Euler to solve a differential equation characterizing elasticity, but nonetheless received a prize from the Paris Academy of Sciences. If q is a safe prime, then it is associated to a prime (q-1)/2, named after this mathematician, who also proved Fermat's Last Theorem in the case of those primes. This mathematician names an identity for factoring x^4+4y^4. For the point, name this 19th century French mathematician.
Answer: Sophie **Germain**

6. This academic has a marriage certificate indicating the profession "theoretical physicist," and once supposedly remarked that "Object-oriented programming is an exceptionally bad idea which could only have originated in California." This Dutch computer scientist's most famous contribution came while pondering at a cafe how to reach Groningen from Rotterdam. For the point, give this namesake of a classical shortest path algorithm.
Answer: Edsger **Dijkstra** ["DIKE-stra"]

7. Former Princeton graduate student Robert Hofstadter is one recipient of this award, as is Frank Wilczek for his work on asymptotic freedom. Developments in topological phases led to Princeton Professor Duncan Haldane sharing this award in 2016, and former Princeton graduate student Kip Thorne shared this award in 2017 for his work on LIGO. For the point, name this annual Swedish prize in a certain science.
Answer: **Nobel Prize in Physics**, prompt on "Nobel Prize"

8. The *Annals* is one of these publications jointly published by Princeton and the Institute for Advanced Study. Before reaching one of these destinations, a paper may first appear as a preprint on the arXiv ["archive"]. For the point, name this type of publication which features articles by professional mathematicians.
Answer: **mathematics journal**

9. Special types of these mathematical objects are named after Stefan Banach and David Hilbert, and in general are subject to only nine axioms. This mathematical object is a module with a group structure. Examples include the set of polynomials, but not the set of polynomials of fixed degree and nonzero leading coefficients. Matrices represent linear operations on these mathematical objects, which can have bases and dimension. For the point, name this type of mathematical object whose elements have magnitude and direction.
Answer: **vector space**, do not accept or prompt on just "vector"

10. One result by this name is sometimes co-named for Henri Cartan, and another is sometimes co-named for Lord Kelvin. The former theorem with this name is a generalization of the Fundamental Theorem of Calculus, and the latter gives a relation involving the curl of a vector field. For the point, give this name for two results in vector calculus, one of which is a special case of the "generalized" other.
Answer: **Stokes'** Theorem

11. One person with this title delegated negotiation of the Convention of Klosterzeven, and a cannonball once struck a portrait hanging in Nassau Hall of that same person with this role. Another person with this role passed a Bill of Rights and was going to be a namesake of Princeton but was already a namesake of another school in Virginia; that other school is the College of William and Mary. For the point, name this monarch of a European polity.
Answer: **King of England**, accept **monarch of England** before mention, do not accept or prompt on another polity or "Queen of England"

12. The naive approach to this task is $O(2^n)$, while a more refined version is $O(2^{\sqrt{n}})$, but since n is exponential in the input, it is still unknown whether this task is feasible in polynomial time, and the most efficient approach currently uses a general field number sieve. This task has been accomplished for RSA-768. Shor's algorithm efficiently achieves this task on a quantum computer. For the point, name this task that involves writing an integer as the product of its prime factors.
Answer: **integer factorization**

13. Some of the earliest instances of these written works are found in the Babylonian *Diagnostic Handbook*, and Thales was the earliest known geometer to compose them. In the 20th century, Brouwer argued the intuitionist viewpoint for these. One method for this is infinite descent, and another is induction, while they can also be depicted in two column. For the point, name this type of argument that unequivocally shows that a result is true.
Answer: **proof**

14. Cayley's theorem ensures that any subgroup is isomorphic to a subgroup of these types of functions. The group of these is denoted S_n, the symmetric group on n letters, and they can be notated using cycle notation. A common problem solving method involving computing the number of these is known as sticks and stones. One of these functions with order two is known as an inversion, and one with no fixed point is a derangement. With n letters, there are n! [n factorial] of these. For the point, name this term for a rearrangement of a list.
Answer: **permutation**, prompt on any synonym

15. This sometime banker studied under Isaac Barrow, and about a tenth of this scientist's written works concern alchemy. This first person to complete the Bernoulli's brachistochrone problem is buried in Westminster Abbey. The texts *Opticks* and *Principia Mathematica* were written, for the point, by what English discoverer of the laws of gravitational motion and cofounder of the infinitesimal calculus?
Answer: Isaac **Newton**

TIEBREAKER: What is the product of the number of recipients of the Fields Medal, Abel Prize, and Wolf Prize currently on the Princeton or IAS faculty?
Answer: 7 * 3 * 6 = 126