Fall 2025 **Math 53, Sections 104/106**

Discussion #17

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1. $\iint_D x\cos y\,dA$ where D is bounded by $y=0, y=x^2, x=1$;

   $\int_0^1\int_0^{x^2} x\cos y dydx = \int_0^1 x\sin y|_0^{x^2}dx = \int_0^1 x\sin x^2 dx = -\frac{1}{2}\cos(x^2)|_0^1 = \frac{1}{2}(1-\cos 1)$.

2. $\iint_D 2x - y\,dA$ where D is bounded by the circle with center at the origin and radius 2.

   $\int_{-2}^2\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} 2x - ydydx = \int_{-2}^2 2xy - \frac{y^2}{2}|_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}dx = \int_{-2}^2 4x\sqrt{4-x^2} = 0$ since $4x\sqrt{4-x^2}$ is an odd function.

3. Find the volume of the solid under the surface $z = xy$ and above the triangle with vertices $(1,1), (4,1), (1,2)$.

   After finding the equations bounding this triangle, we set up the integral and solve. The bounding lines are $x=1, y=1, x+3y=7$ so the volume is

   $$\int_1^2\int_1^{7-3y} xydxdy = \int_1^2 \frac{1}{2}x^2y|_1^{7-3y}dy = \frac{1}{2}\int_1^2 48y - 42y^2 + 9y^3dy = \frac{31}{8}$$

4. Find the volume enclosed by $z=x^2, y=x^2$ and the planes $z=0$ and $y=4$.

   The set up is $\int_{-2}^2\int_{x^2}^4 x^2dydx = \int_{-2}^2 4x^2 - x^4dx = \frac{128}{15}$.

5. Find the volume of the solid by subtracting two volumes. The solid is enclosed by $y = 1-x^2, y = x^2-1$, and the planes $x+y+z=2, 2x+2y-z+10=0$.

   The region of integration is bounded by the curves $y=1-x^2, y=x^2-1$ which intersect at $x=\pm 1$ with $1-x^2 \geq x^2-1$ on $[-1,1]$. Within this region, $z = 2x+2y+10$ is above $z = 2-x-y$ so we have

   $$V = \int_{-1}^1\int_{x^2-1}^{1-x^2} 2x+2xy+10dydx - \int_{-1}^1\int_{x^2-1}^{1-x^2} 2-x-ydydx = \int_{-1}^1\int_{x^2-1}^{1-x^2} 3x+3y+8dydx$$
   $$= \int_{-1}^1 3xy + \frac{3}{2}y^2 + 8y|_{x^2-1}^{1-x^2}dx = \int_{-1}^1 -6x^3 - 16x^2 + 6x + 16dx = \frac{64}{3}.$$

All problems courtesy of Carlos Esparza.