Fall 2025 **Math 53, Sections 104/106**

Discussion #11

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1. What are the extreme values of $f(x,y) = x^2 - 10xy + y^2$ on all of $\mathbb{R}^2$? Try to find its extrema using the gradient vector and the discriminant D. Think about the answer in relation to the values of f_{xx} and f_{yy}.

2. Find the local maximum and minimum values and saddle point(s) of the following functions.

   (a) $f(x,y) = x^2 + y^4 + 2xy$

   (b) $f(x,y) = xy + e^{-xy}$

   (c) $f(x,y) = y\sin(\pi x)$

3. Find the global maxima and minima of the following functions on their indicated domains.

   (a) The function $f(x,y) = x^2 - y$ on the domain $D = [0,2] \times [0,2]$.

   (b) The function $f(x,y) = x - y$ on the domain $\{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$.

   (c) The function $f(x,y) = x^2 - xy + y^2 - 3y$ on the region bounded by the x and y axes and the line $x + y = 4$.

   (d) The function $f(x,y) = \sin x \cdot \sin y$ on the domain $-1 < x < \pi$ and $-\pi < y < \pi$.