Fall 2025 **Math 53, Sections 104/106**

Discussion #9

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1. Do directional derivatives commute? i.e., for unit vectors $\boldsymbol{u}$ and $\boldsymbol{v}$, and twice-partially-differentiable f (with any number of inputs; you can assume 2), is it the case that $D_{\boldsymbol{v}} D_{\boldsymbol{u}} f = D_{\boldsymbol{u}} D_{\boldsymbol{v}} f$? Either prove it or provide a counterexample.

2. Suppose the following are true:

$$D_{\left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle} f = e^x(\sin y + \cos y)$$
$$D_{\left\langle \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right\rangle} f = e^x(-\sin y + \cos y).$$

   Find ∇f.

3. Compute the following tangent planes:

   (a) $f(x,y,z) = d$, for $f(x,y,z) = ax + by + cz$, at any point (x_0, y_0, z_0). (Do this one with partial derivatives. Could you have done this another way?)

   (b) $xy^2z^3 = 8$ at $(2,2,1)$;

   (c) $x + y + z = e^{xyz}$ at $(0,0,1)$.

   (d) Show that the equation of the tangent plane to the ellipsoid $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$ at the point (x_0, y_0, z_0) can be written as

$$\frac{xx_0}{a^2} + \frac{yy_0}{b^2} + \frac{zz_0}{c^2} = 1.$$

4. (*This one is pretty tricky*) Consider the spheres $\left(x - \frac{1}{\sqrt{2}}\right)^2 + y^2 + z^2 = 1$ and $\left(x + \frac{1}{\sqrt{2}}\right)^2 + y^2 + z^2 = 1$. For each point of their intersection, find the angle between the tangent plane to each sphere.