Math 53, Fall 2025, Section 104, Quiz 5

Name: Solutions Student ID: ______________

Time limit: 20 minutes.

- ***Please keep your work only on the two printed pages.***
- ***An answer for problems 1 and 3 without any work shown will get no credit.***

If a problem asks for a specific answer (rather than an explanation), box your result. You do not need to simplify expressions such as $2(x-1)+x$, but you should evaluate trigonometric functions of simple angles such as multiples of $\frac{\pi}{4}$ and $\frac{\pi}{6}$.

1. **(3 points each)** Let $\vec{F} = \langle y^2z, 2xyz, 0\rangle$ and $f = -xy^2z$. For each of the following expressions: either evaluate it if possible, or write "IDNMS" (*it does not make sense*).

   (a) $\nabla \times f$ $\boxed{\text{IDNMS}}$

   (b) $\nabla \cdot (\nabla \times \vec{F})$ $\boxed{0}$

   (c) $\nabla \cdot (\nabla \cdot \vec{F})$ $\boxed{\text{IDNMS}}$

   (d) $\vec{F} + \nabla f$ $\boxed{\langle 0, 0, -xy^2\rangle}$

2. **(10 points)** Compute the area of the plane $2x+4y+4z = 0$ lying inside the cylinder $y^2+z^2 = 1$.

It's the graph of $x = -2y-2z$

$$r(y,z) = (-2y-2z, y, z)$$

$$r_y(y,z) = \langle -2, 1, 0\rangle$$

$$r_z(y,z) = \langle -2, 0, 1\rangle$$

$$n = r_y \times r_z = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{vmatrix} = \langle 1, 2, 2\rangle$$

$$|n| = 3$$

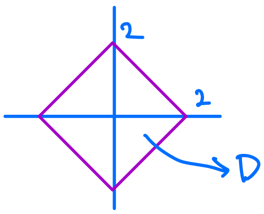
Let D be the unit disk in the yz plane.

$$\text{Answer} = \iint_D |n|\, dA = 3\ \text{area}(D) = \boxed{3\pi}$$

3. **(10 points)** Evaluate the following line integral, where C is a counterclockwise traversal of the boundary of the square with vertices $(-2,0)$, $(0,-2)$, $(2,0)$, and $(0,2)$.

$$\int_C \left(xy + e^{-x^2}\right) \mathrm{d}x + (x + y + \sin y)\,\mathrm{d}y.$$

(*Hint: draw the domain of integration and look for symmetries.*)



Green's thm: $= \iint_D 1 - x \, dA = \text{area}(D) - \iint_D x \, dA$

$= (2\sqrt{2})^2 - 0$ by symmetry of D about y-axis.

$= \boxed{8}$

Can also evaluate $\iint_D x\,dA = \int_{-2}^{2} \int_{|x|-2}^{2-|x|} x \, dy\,dx$

$= 2\int_{-2}^{2} x(2-|x|)\,dx$

$= 4\int_{-2}^{2} x\,dx - 2\left(\int_{0}^{2} x^2\,dx - \int_{-2}^{0} x^2\,dx\right)$

$= 2x^2\Big|_{-2}^{2} - \frac{2}{3}x^3\Big|_{0}^{2} + \frac{2}{3}x^3\Big|_{-2}^{0}$

$= 0 - \frac{16}{3} + \frac{16}{3}$

$= 0$