Fall 2025 **Math 53, Sections 104/106**

Discussion #7

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Date: September 17

1. For each of the following, determine whether the limit exists.

   (a) $f(x,y) = \alpha x + \beta y + \gamma$ and $(x,y) \to (a,b)$. (Try doing this one with ε–δ.)

   (b) $f(x,y) = xy \sin \dfrac{1}{x^2+y^2}$ and $(x,y) \to (0,0)$.

   (c) $f(x,y) = \dfrac{(x-y)^2(x+y)}{(x-y)^4+(x+y)^2}$ and $(x,y) \to (0,0)$.

2. Find the best linear approximation to each of the following functions near the corresponding input values.

   (a) $f(x,y) = y^2 - x$ near the input $(3,0)$.

   (b) $g(x,y) = e^x \cos y$ near the input $(5, \pi/2)$.

   (c) $h(x,y,z) = xyz$ near the input $(3,0,2)$.

   (d) $p(x,y,z,w) = x^2+y^2+z^2+w^2$ near the input $(0,1,0,-1)$.

3. Compute the gradients of the following functions.

   (a) $f(\theta, \phi) = \cos\theta \cos\phi$.

   (b) $f(x, y) = \arctan(y/x)$.

   (c) $f(t, x, y) = \frac{1}{\sqrt{4\pi t}} \exp(-(x-y)^2/4t))$. (Express everything as multiples of $f(t, x, y)$.)

4. Consider $f(x, y) = e^{-r^4}$ where $r = \sqrt{x^2 + y^2}$. Compute its directional derivative at $(0, 0)$ w.r.t. the unit vectors in (polar) directions $\theta = 0, \pi/4, \pi/2$. What about any other angle?