Math 53, Fall 2025, Section 104, Quiz 4

Name: _____ Student ID: _____

Time limit: 20 minutes.

- ***Please keep your work only on the two printed pages.***
- ***An answer without any work shown will get no credit.***

Each of the three problems is worth 10 points. If a problem asks for a specific answer (rather than an explanation), box your result. You do not need to simplify expressions such as $2(x-1)+x$, but you should evaluate trigonometric functions of simple angles such as multiples of $\frac{\pi}{4}$ and $\frac{\pi}{6}$.

1. Compute the Jacobian (i.e., the absolute value of the determinant of the Jacobian matrix) for the following changes of variables.

   (a) $x = -u + 3v$ and $y = u - 2v$, for u and v both in the interval $[0, 1]$.

   (b) $x = u^2$, $y = v^2 - uw$, and $z = w^2$, for u, v, and w all in the interval $[-1, 0]$.

2. Compute the volume of the solid inside the sphere $x^2 + y^2 + z^2 = 9$ and outside the cylinder $x^2 + y^2 = 4$.

3. Let E be the solid inside the sphere $x^2 + y^2 + z^2 = 2$ with $z > 0$. Compute

$$\iiint_E z \, \mathrm{d}V.$$