Fall 2025 **Math 53, Sections 104/106**

Discussion #5

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1. For each of the following functions, find their first derivatives, i.e. $f_x(x,y)$ and $f_y(x,y)$, and also compute $f_x(1,1)$ and $f_y(1,1)$.

   (a) $f(x,y) = x^4 + 5xy^3$

   (b) $f(x,y) = \frac{x}{y}$

   (c) $f(x,y) = x\sin(x+y) + y^2$

2. Calculate the first derivatives, i.e. f_x, f_y, and f_z, for the function

$$f(x,y,z) = \cos(x^2+2y) - e^{4x-z^4y} + y^3$$

3. Find the tangent planes to the graphs of each of the following functions at an arbitrary point $(x_0, y_0, f(x_0,y_0))$.

   (a) $f(x,y) = x^2 + 2xy + y^2$

   (b) $f(x,y) = e^{xy}$.

   (c) $f(x,y) = \sin x$.

4. (a) Find $\frac{dz}{dt}$ for $z = xy^3 - x^2y$, $x = t^2 + 1$, $y = t^2 - 1$.

   (b) Find $\frac{dw}{dt}$ if $w = f(x, y, z) = xe^{y/z}$, if $x = t^2$, $y = 1 - t$, and $z = 1 + 2t$.

   (c) $w = xy + yz + zx$, $x = r\cos\theta$, $y = r\sin\theta$, $z = r\theta$, find the derivatives $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial \theta}$ when $r = 2$ and $\theta = \frac{\pi}{2}$

5. (a) Use implicit differentiation to find $\frac{dy}{dx}$ for $y\cos x = x^2 + y^2$

   (b) Use implicit differentiation to find $\frac{\partial z}{\partial y}$ and $\frac{\partial z}{\partial x}$ for the equation $e^z = xyz$.