Fall 2025 **Math 53, Sections 104/106**

Discussion #4

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1. Describe the level sets of the following functions:

   (a) $f(x) = \tan x$.

   (b) $f(x, y) = \tan x$.

   (c) $f(x, y) = \ln(x - y) + \ln(x - 5)$. What is the union of the level sets here?

   (d) $f(x, y, z) = x^2 + 2y^2 + 3z^2$.

   (a) $f(x) = c$ when $x = \arctan c + \pi k$ for $k \in \mathbb{Z}$ (recall that arctan is defined to lie between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$). So each level set is an equally-spaced set of points in $\mathbb{R}$.

   (b) $f(x, y) = c$ when $x = \arctan c + \pi k$ for $k \in \mathbb{Z}$ and y is anything. So each level set is an equally-spaced set of lines in $\mathbb{R}^2$.

   (c) $f(x, y) = c$ when $(x - y)(x - 5) = e^c$, or $y = x - \frac{e^c}{x-5}$. However, this only makes sense when $x > y$ and $x > 5$ in order for the logarithms to be defined. Thus, this is also the domain of the function.

   (d) The level sets are $x^2 + 2y^2 + 3z^2 = c$ (for $c > 0$). But what are these? They're ellipsoids—stretched spheres—and they can themselves be visualized via their own level sets, which are ellipses.

2. What is the velocity of a particle moving along $r(t) = (3t, 4 - t^2, \sin t)$ when $t = 0$? What about the speed? What if instead the velocity was $v(t) = \langle 3t, 4 - t^2, \sin t\rangle$ with initial position at the origin, and we want to know the position at $t = 1$?

   $r'(t) = \langle 3, 0, \cos t\rangle$ so $r'(0) = \langle 3, 0, 1\rangle$ and $\|r'(0)\| = \sqrt{10}$. $\int v(t)\,\mathrm{d}t = \left\langle \frac{3}{2}t^2, 4t - \frac{t^3}{3}, -\cos t\right\rangle$ so when $t = 1$ the position is $\left(\frac{3}{2}, \frac{11}{3}, -1\right)$.

3. Write down an explicit tangent line for each point of the curve $r(t) = (4t, 5 - t^3, \sin t)$.

   At time t, the tangent vector at $r(t)$ is $r'(t) = \langle 4, -3t^2, \cos t\rangle$ so the tangent line is $\ell(s) = \left(4t + 4s, 4 - t^3 - 4t^2 s, \sin t + s\cos t\right)$ for $s \in \mathbb{R}$.

4. Parameterize the intersection of the surfaces $z = x^2 + y^2$ and $y + z = 1$ in $\mathbb{R}^3$.

   $z = 1 - y$ so $x^2 + y^2 = 1 - y$ or $x^2 + \left(y + \frac{1}{2}\right)^2 = \frac{5}{4}$. If $y + \frac{1}{2} = \frac{\sqrt{5}}{2}\sin t$ then $x = \frac{\sqrt{5}}{2}\cos t$ (or $x = -\frac{\sqrt{5}}{2}\cos t$) and we have $(x, y, z) = \left(\frac{\sqrt{5}}{2}\cos t, \frac{\sqrt{5}}{2}\sin t - \frac{1}{2}, \frac{3}{2} - \frac{\sqrt{5}}{2}\sin t\right)$.