Math 53, Fall 2025, Section 106, Quiz 3

Name: _____ Student ID: _____

Time limit: 20 minutes.

- ***Please keep your work only on the two printed pages.***
- ***An answer without any work shown will get no credit.***

If a problem asks for a specific answer (rather than an explanation), box your result. You do not need to simplify expressions such as $2(x-1)+x$, but you should evaluate trigonometric functions of simple angles such as multiples of $\frac{\pi}{4}$ and $\frac{\pi}{6}$.

1. (5 points each, **part (b) is extra credit**) For each region R, write the area integral $\iint_R \mathrm{d}A$ as two equivalent double integrals: one in terms of $\mathrm{d}x\,\mathrm{d}y$ and one in terms of $\mathrm{d}y\,\mathrm{d}x$. You ***do not*** need to evaluate the integrals. Answers with variables out of scope (as in $\int_x^1 \int_1^0 \mathrm{d}x\,\mathrm{d}y$) will receive little or no credit. (*Hint: drawing the regions will help!*)

   (a) R is the triangle with vertices $(5,6)$, $(6,6)$, and $(6,7)$.

   (b) *Your answer to this one cannot be a sum of integrals.* R is the triangle with vertices $(0,0)$, $(3,3)$, $(-3,3)$.

2. (5 points each) Evaluate each of the following integrals.

   (a) $\int_2^6 \int_5^7 \left(\frac{1}{x}+xy\right) \mathrm{d}y\,\mathrm{d}x$. (b) $\int_{-1}^0 \int_y^{y^2} y\,\mathrm{d}x\,\mathrm{d}y$.

3. (10 points) Use Lagrange multipliers to find the distance between the point $(-2, 1, 2)$ and the nearest point on the surface $x^2 + y^2 + z^2 = 16$, as well as the location of that point (or points). **Extra credit: find a solution that doesn't use derivatives at all.**