# Math 53, Fall 2025, Section 104, Quiz 3

Name: Solutions Student ID: ____________

Time limit: 20 minutes.

- ***Please keep your work only on the two printed pages.***
- ***An answer without any work shown will get no credit.***

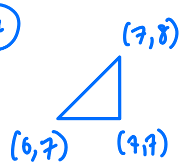
If a problem asks for a specific answer (rather than an explanation), box your result. You do not need to simplify expressions such as $2(x-1)+x$, but you should evaluate trigonometric functions of simple angles such as multiples of $\frac{\pi}{4}$ and $\frac{\pi}{6}$.

1. (5 points each, **part (b) is extra credit**) For each region $R$, write the area integral $\iint_R \mathrm{d}A$ as two equivalent double integrals: one in terms of $\mathrm{d}x\,\mathrm{d}y$ and one in terms of $\mathrm{d}y\,\mathrm{d}x$. You ***do not*** need to evaluate the integrals. Answers with variables out of scope (as in $\int_x^1 \int_1^0 \mathrm{d}x\,\mathrm{d}y$) will receive little or no credit. (*Hint: drawing the regions will help!*)

   (a) $R$ is the triangle with vertices $(6,7)$, $(7,7)$, and $(7,8)$.

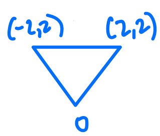
   (b) *Your answer to this one cannot be a sum of integrals.* $R$ is the triangle with vertices $(0,0)$, $(2,2)$, $(-2,2)$.

(a)




$$\boxed{\int_6^7 \int_7^{x+1} dy\,dx = \int_7^8 \int_{y-1}^7 dx\,dy}$$

(b)




$$\boxed{\int_{-2}^2 \int_{|x|}^2 dy\,dx = \int_0^2 \int_{-y}^y dx\,dy}$$

2. (5 points each) Evaluate each of the following integrals.

   (a) $\int_3^6 \int_5^7 \left(\frac{1}{x} + xy\right) \mathrm{d}y\,\mathrm{d}x$. (b) $\int_{-1}^0 \int_y^{y^2} y\,\mathrm{d}x\,\mathrm{d}y$.

(a)
$$= \int_3^6 \frac{2}{x} + \frac{1}{2}xy^2\Big|_5^7 \, dx$$
$$= \int_3^6 \frac{2}{x} + 12x\,dx$$
$$= 2\ln 2 + 6x^2\Big|_3^6$$
$$= \boxed{2\ln 2 + 162}$$

(b)
$$= \int_{-1}^0 y(y^2 - y)\,dy$$
$$= \int_{-1}^0 y^3 - y^2\,dy$$
$$= \frac{1}{4}y^4 - \frac{1}{3}y^3\Big|_{-1}^0$$
$$= -\frac{1}{4} - \frac{1}{3} = \boxed{-\frac{7}{12}}$$

3. (10 points) Use Lagrange multipliers to find the distance between the point $(1, 2, -2)$ and the nearest point on the surface $x^2 + y^2 + z^2 = 16$, as well as the location of that point (or points). **Extra credit: find a solution that doesn't derivatives at all.**

g

use

$f(x,y,z) = \text{distance}^2$

$= (x-1)^2 + (y-2)^2 + (z+2)^2$

$\nabla f(x,y,z) = \langle 2(x-1), 2(y-2), 2(z+2) \rangle$

$\nabla g(x,y,z) = \langle 2x, 2y, 2z \rangle$

$\nabla f = \lambda \nabla g$ :

$$\left.\begin{array}{l} x-1 = \lambda x \Rightarrow x = \frac{1}{1-\lambda} \\ y-2 = \lambda y \Rightarrow y = \frac{2}{1-\lambda} \\ z+2 = \lambda z \Rightarrow z = \frac{-2}{1-\lambda} \end{array}\right\} \text{ or } \lambda = 1. \Downarrow -1 = 0. \ \underline{\text{No}}$$

Plug in to $g = 16$:

$$\frac{9}{(1-\lambda)^2} = 16, \quad 1-\lambda = \pm 3/4$$

$$\lambda = 1/4 \text{ or } 7/4$$

$\lambda = 1/4$: $\left(\frac{4}{3}, \frac{8}{3}, -\frac{8}{3}\right)$ dist $= \sqrt{f} = 1$

$\lambda = 7/4$: $\left(-\frac{4}{3}, \frac{8}{3}, -\frac{8}{3}\right)$ dist $= 7$

$$\boxed{\begin{array}{l} \text{distance} = 1 \\ \text{point} = \left(\frac{4}{3}, \frac{8}{3}, -\frac{8}{3}\right) \end{array}}$$

alt. sol'n: by symmetry, answer should be the nearer end of the point's diameter. $\Rightarrow$ scale by $\frac{4 \rightarrow \text{radius of sphere}}{3 \rightarrow \text{distance from origin}}$.

to get $\left(\frac{4}{3}, \frac{8}{3}, -\frac{8}{3}\right)$

distance is $4 - 3 = 1$.