Math 53, Fall 2025, Section 106, Quiz 2

Name: Solutions Student ID: ____________

Time limit: 20 minutes. Each of the three problems is worth 10 points. If a problem asks for a specific answer (rather than an explanation), box your result. ***An answer without any work shown will get no credit.*** You do not need to simplify expressions such as $2(x-1)+x$, but you should evaluate trigonometric functions of simple angles such as multiples of $\frac{\pi}{4}$ and $\frac{\pi}{6}$.

1. Each of the following limits does not exist. Why? *Read each limit carefully!* **Part (c) is extra credit.**

(a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2+y^3}{x^2+4y^3}$ (b) $\displaystyle\lim_{(x,y)\to(7,7)} \frac{x-7}{x}\cdot\frac{y}{y-7}$ (c) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\sin\frac{1}{x}}{\sin\frac{1}{y}}$

(a) $x=0,\ y\neq 0:\ 1/4$

$x\neq 0,\ y=0:\ 1$

(b) $x=y:\ \frac{x-7}{x}\frac{y}{y-7}=1$

$x=7, y\neq 7:\ 0$

(c) $x=\frac{1}{\frac{\pi}{2}+2\pi n},\ y=\frac{1}{\pi n}:$

at these points going to 0, the function is undefined.

I did not initially see this very clean solution, but there were multiple submissions that found it!

2. Suppose f is a function of two real variables and has continuous partial derivatives, that is, f is "nice." Let $z = f(3p^2+q, p+q) - f(2p^2+q, p+2q)$. Express $\frac{\partial^2 z}{\partial q^2}$ in terms of p, q, and the partial derivatives of f. (i.e., x and y should not appear except in ∂_x and ∂_y or f_x and f_y.)

$$w=\frac{\partial z}{\partial q} = \underbrace{f_x}_{g}(3p^2+q, p+q) + \underbrace{f_y}_{h}(3p^2+q, p+q) - f_x(2p^2+q, p+2q) - 2f_y(2p^2+q, p+2q)$$

$$\frac{\partial^2 z}{\partial q} = \frac{\partial w}{\partial q} = g_x(3p^2+q,p+q) + g_y(3p^2+q,p+q) + h_x(3p^2+q,p+q) + h_y(3p^2+q,p+q)$$
$$- g_x(2p^2+q,p+2q) - 2g_y(2p^2+q,p+2q) - 2h_x(2p^2+q,p+2q) - 4h_y(2p^2+q,p+2q)$$

$$= \boxed{\begin{aligned} &f_{xx}(3p^2+q,p+q) + 2f_{xy}(3p^2+q,p+q) + f_{yy}(3p^2+q,p+q) \\ &- f_{xx}(2p^2+q,p+2q) - 4f_{xy}(2p^2+q,p+2q) - 4f_{yy}(2p^2+q,p+q). \end{aligned}}$$

3. Consider the surface $(x, y, (x+2y)^2 - 9)$ in $\mathbb{R}^3$, with $-3 < x < y < 3$.

   (a) Find the equation of the tangent plane at the point $(1, 2, 16)$.

   (b) Find two distinct points on the surface whose tangent planes are parallel. (*Hint: recall that parallel planes never intersect. What does that mean about their equations?*)

(a) $z - 16 = f_x(1,2)(x-1) + f_y(1,2)(y-2)$ for $f(x,y) = (x+2y)^2 - 9$

$f_x = 2(x+2y)$, $f_y = 4(x+2y)$

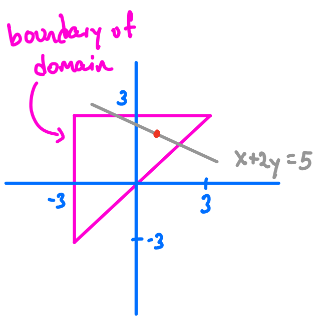
$= 10(x-1) + 20(y-2)$

or, $\boxed{10x + 20y - z = 34}$.

(b) Let's just find a second point w/ normal vector $\langle 10, 20, -1 \rangle$ to its tangent plane.

i.e., we want x', y' s.t. $x' + 2y' = 5$, so that $f_x(x', y') = 10$, $f_y(x', y') = 20$

There are many such points.



One is $(0.5, 2.25)$

So the two points are

$\boxed{(1, 2, 16),\ (0.5, 2.25, 16)}$