

Math 53, Fall 2025, Section 106, Quiz 2

Name: _____

Student ID: _____

Time limit: 20 minutes. Each of the three problems is worth 10 points. If a problem asks for a specific answer (rather than an explanation), box your result. *An answer without any work shown will get no credit.* You do not need to simplify expressions such as $2(x - 1) + x$, but you should evaluate trigonometric functions of simple angles such as multiples of $\frac{\pi}{4}$ and $\frac{\pi}{6}$.

1. Each of the following limits does not exist. Why? *Read each limit carefully!* **Part (c) is extra credit.**

(a) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + y^3}{x^2 + 4y^3}$ (b) $\lim_{(x,y) \rightarrow (7,7)} \frac{x-7}{x} \cdot \frac{y}{y-7}$ (c) $\lim_{(x,y) \rightarrow (0,0)} \frac{\sin \frac{1}{x}}{\sin \frac{1}{y}}$

2. Suppose f is a function of two real variables and has continuous partial derivatives, that is, f is “nice.” Let $z = f(3p^2 + q, p + q) - f(2p^2 + q, p + 2q)$. Express $\frac{\partial^2 z}{\partial q^2}$ in terms of p , q , and the partial derivatives of f . (i.e., x and y should not appear except in ∂_x and ∂_y or f_x and f_y .)

3. Consider the surface $(x, y, (x + 2y)^2 - 9)$ in \mathbb{R}^3 , with $-3 < x < y < 3$.
- (a) Find the equation of the tangent plane at the point $(1, 2, 16)$.
 - (b) Find two distinct points on the surface whose tangent planes are parallel. (*Hint: recall that parallel planes never intersect. What does that mean about their equations?*)