

## Math 53, Fall 2025, Section 106, Quiz 2

Name: \_\_\_\_\_

Student ID: \_\_\_\_\_

Time limit: 20 minutes. Each of the three problems is worth 10 points. If a problem asks for a specific answer (rather than an explanation), box your result. *An answer without any work shown will get no credit.* You do not need to simplify expressions such as  $2(x - 1) + x$ , but you should evaluate trigonometric functions of simple angles such as multiples of  $\frac{\pi}{4}$  and  $\frac{\pi}{6}$ .

1. Each of the following limits does not exist. Why? *Read each limit carefully! Part (c) is extra credit.*

$$(a) \lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + y^3}{x^2 + 4y^3} \quad (b) \lim_{(x,y) \rightarrow (7,7)} \frac{x-7}{x} \cdot \frac{y}{y-7} \quad (c) \lim_{(x,y) \rightarrow (0,0)} \frac{\sin \frac{1}{x}}{\sin \frac{1}{y}}$$

2. Suppose  $f$  is a function of two real variables and has continuous partial derivatives, that is,  $f$  is “nice.” Let  $z = f(3p^2 + q, p + q) - f(2p^2 + q, p + 2q)$ . Express  $\frac{\partial^2 z}{\partial q^2}$  in terms of  $p$ ,  $q$ , and the partial derivatives of  $f$ . (i.e.,  $x$  and  $y$  should not appear except in  $\partial_x$  and  $\partial_y$  or  $f_x$  and  $f_y$ .)

3. Consider the surface  $(x, y, (x + 2y)^2 - 9)$  in  $\mathbb{R}^3$ , with  $-3 < x < y < 3$ .

- (a) Find the equation of the tangent plane at the point  $(1, 2, 16)$ .
- (b) Find two distinct points on the surface whose tangent planes are parallel. (*Hint: recall that parallel planes never intersect. What does that mean about their equations?*)