# Math 53, Fall 2025, Section 104, Quiz 2

Name: _____ Student ID: _____

Time limit: 20 minutes. Each of the three problems is worth 10 points. If a problem asks for a specific answer (rather than an explanation), box your result. ***An answer without any work shown will get no credit.*** You do not need to simplify expressions such as $2(x-1)+x$, but you should evaluate trigonometric functions of simple angles such as multiples of $\frac{\pi}{4}$ and $\frac{\pi}{6}$.

1. Each of the following limits does not exist. Why? *Read each limit carefully!* **Part (c) is extra credit.**

   (a) $\lim_{(x,y)\to(0,0)} \dfrac{x^2+y^3}{2x^2+y^3}$ (b) $\lim_{(x,y)\to(5,5)} \dfrac{x-5}{x} \cdot \dfrac{y}{y-5}$ (c) $\lim_{(x,y)\to(0,0)} \dfrac{\cos\frac{1}{x}}{\cos\frac{1}{y}}$

2. Suppose $f$ is a function of two real variables and has continuous partial derivatives, that is, $f$ is "nice." Let $z = f(3p^2+q, p+3q) - f(2p^2+q, p+q)$. Express $\dfrac{\partial^2 z}{\partial q^2}$ in terms of $p$, $q$, and the partial derivatives of $f$. (i.e., $x$ and $y$ should not appear except in $\partial_x$ and $\partial_y$ or $f_x$ and $f_y$.)

3. Consider the surface $\big(x, y, (2x+y)^2 - 4\big)$ in $\mathbb{R}^3$, with $-3 < x < y < 3$.

   (a) Find the equation of the tangent plane at the point $(1, 2, 12)$.

   (b) Find two distinct points on the surface whose tangent planes are parallel. (*Hint: recall that parallel planes never intersect. What does that mean about their equations?*)