Fall 2025 **Math 53, Sections 104/106**

Discussion #2/3

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Date: September 5/8

1. Write equations in polar coordinates to describe the following curves. Make sure to include the range for θ.
   (a) The curve $xy = 1$ for $x > 0$.
   (b) The parabola $x = y^2$.
   (c) The line $x = 1$.
2. Consider the polar curve $r = 2\cos\theta$ for $0 \leqslant \theta \leqslant 2\pi$. Verify that it describes a circle centered at $(1, 0)$ with radius 1. How many times does it wrap around?
3. (a) Find the area enclosed by the curve $x = t^2 - 2t$, $y = \sqrt{t}$ and the y-axis.
   (b) Find the area enclosed by the x-axis and the curve $x = t^3 + 1$, $y = 2t - t^2$.
4. Find the area of the region that lies inside $r = 3\cos\theta$ and outside $r = 1 + \cos\theta$.

5. Find the length of each curve:

   (a) $r = 2\cos\theta, 0 \leq \theta \leq \pi$.

   (b) $r = \theta^2, 0 \leq \theta \leq 2\pi$.

6. (a) If $\vec{u}$ and $\vec{v}$ are unit vectors in $\mathbb{R}^3$ and $u \circ v = -1$, what is the angle between $\vec{u}$ and $\vec{v}$?

   (b) Find three nonzero vectors in $\mathbb{R}^3$ that are perpendicular to $\langle 1, 3, 2 \rangle$.

   (c) Let P be a vertex on a cube. Let Q be an adjacent vertex and let R be the vertex opposite to P. Using dot products, find the angle between the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

   (d) If $\vec{u}$ and $\vec{v}$ are unit vectors in $\mathbb{R}^3$, show that the vectors $\vec{u} + \vec{v}$ and $\vec{v} - \vec{v}$ are perpendicular.

   (e) Find the vector projection of $\vec{v}$ onto $\vec{w}$ and the scalar projection of $\vec{v}$ onto $\vec{w}$ if $\vec{v} = \langle 2, 4 \rangle$, $\vec{w} = \langle 3, 1 \rangle$.

7. (a) Find the cross products $\vec{v} \times \vec{w}$ if $\vec{v} = \langle 2, 3, 1 \rangle$ and $\vec{w} = \langle -1, 2, 3 \rangle$.

   (b) Let $\vec{u}$ and $\vec{v}$ be nonzero vectors with $\vec{u} \times \vec{v} = \vec{0}$. What can you say about the relationship between $\vec{u}$ and $\vec{v}$?

   (c) Find the area of the triangle with two sides given by the vectors $\vec{v} = \langle 1, 2 \rangle$ and $\vec{w} = \langle -3, 4 \rangle$.