Fall 2025 **Math 53, Sections 104/106**

Discussion #32/33

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1. Evaluate the surface integral $\iint_S \vec{F} \cdot \mathrm{d}\vec{S}$ for the given vector field $\vec{F}$ and oriented surface S. For closed surfaces, use the positive (outward) orientation.

   (a) $\vec{F}(x,y,z) = \langle ze^{xy}, -3ze^{xy}, xy\rangle$. S is the parallelogram $x = u+v, y = u-v, z = 1+2u+v, 0 \leqslant u \leqslant 2, 0 \leqslant v \leqslant 1$ oriented upwards.

   (b) $\vec{F}(x,y,z) = \langle 0, y, -z\rangle$ and S consists of the paraboloid $y = x^2+z^2, 0 \leqslant y \leqslant 1$, and the disk $x^2+z^2 \leqslant 1, y = 1$.

   (c) $\vec{F}(x,y,z) = \langle x^2, y^2, z^2\rangle$ and S is the boundary of the solid half cylinder $0 \leqslant z \leqslant \sqrt{1-y^2}, 0 \leqslant x \leqslant 2$.

2. Let S be the cylinder $x^2+y^2 = 1$, $-1 \leqslant z \leqslant 1$, plus its top and bottom caps. Compute the flux of the vector field

$$\vec{F}(x,y,z) = \begin{pmatrix} -\sin \pi y \\ -\cos \pi x \\ xy \end{pmatrix}$$

   both directly and by using the divergence theorem.

3. (a) Compute $\iint_S \vec{F} \cdot \mathrm{d}\vec{S}$ where $\vec{F} = (x^2, 2z, -3y)$ and S is the portion of $y^2 + z^2 = 4$ between $x = 0$ and $x = 3 - z$.

   (b) Compute $\iint_S (\nabla \times \vec{F}) \cdot \mathrm{d}\vec{S}$ where $\vec{F} = (y, -x, yx^3)$ and S is the portion of the sphere of radius 4 with $z \geq 0$ and the upwards orientation.

   (c) Compute $\iint_S \vec{F} \cdot \mathrm{d}\vec{S}$ where $\vec{F} = (\sin(\pi x), zy^3, z^2 + 4x)$ where S is the surface of the box $-1 \leqslant x \leqslant 2$, $0 \leqslant y \leqslant 1$, and $1 \leqslant z \leqslant 4$, oriented outwards.

4. Let $\vec{F}(x, y, z) = (x^2, yz, xz)$ and evaluate $\displaystyle\iint_S \nabla \times \vec{F} \cdot \mathrm{d}\vec{S}$, where S is the unit sphere...

   - by direct computation.
   - using a symmetry argument.
   - using the divergence theorem.
   - using Stokes' theorem.