Fall 2025 **Math 53, Sections 104/106**

Discussion #23

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1. For each of the following vector fields $\vec{F}$, either prove that $\vec{F}$ is conservative by finding a function f such that $\nabla f = \vec{F}$, or prove that f is not conservative.

   (a) $\vec{F}(x, y) = x\,\vec{i} + y\,\vec{j}$
   (b) $\vec{F}(x, y) = x\,\vec{i} + x\,\vec{j}$
   (c) $\vec{F}(x, y, z) = yz\,\vec{i} + xz\,\vec{j} + xy\,\vec{k}$
   (d) $\vec{F} = xz\,\vec{i} + yz\,\vec{j} + xy\,\vec{k}$

2. Compute the following line integrals:

   (a) $\int_C x\,\mathrm{d}s$ where C is the graph of $f(x) = \frac{1}{2}x^2$ going from $x = 0$ to $x = 2$.
   (b) $\int_C xy^4\,\mathrm{d}s$ where C is the right half of the unit circle.
   (c) $\int_C x^2y\,\mathrm{d}s$ in 3D where C is given by $x = \cos t, y = \sin t, z = t, 0 \leq t \leq \pi/2$.

3. Compute the following line integrals:

   (a) $\int_C y^2\,\mathrm{d}x + x^2\,\mathrm{d}y$ where C is the line segment from $(1, 0)$ to $(4, 1)$.
   (b) $\int_C x\,\mathrm{d}x + y\,\mathrm{d}y + z\,\mathrm{d}z$ where C is the straight line connecting $(0, 0, 0)$ to $(1, 2, 3)$. Can you figure out what the integral will be when the endpoint of C is an arbitrary point (x_0, y_0, z_0) ?
   (c) $\int_C \vec{F} \cdot \mathrm{d}\vec{r}$ where $\vec{F}(x, y) = (y, 1)$ and C is the unit circle, traversed counterclockwise. Can you say something about the integral of $\vec{F}_2(x, y) = (y, 0)$ along the same curve without doing another computation?

All problems courtesy of Carlos Esparza.