Math 53, Fall 2025, Section 106, Quiz 1

Name: _____ Date: _____

Time limit: 20 minutes. Each of the three problems is worth 10 points. If a problem asks for a numerical answer, please box your result. An answer without any work shown will get no credit. You do not need to simplify expressions such as $2(x-1)+(x-4)$, but you should evaluate trigonometric functions of simple angles such as multiples of $\frac{\pi}{4}$ and $\frac{\pi}{6}$.

1. Consider the curve $r = 7\sin\theta$.

   (a) As θ ranges across all of $\mathbb{R}$, the curve is traced out many times. Find a range of θ such that the curve is traced out exactly once.

   (b) By taking an integral, find the length of that single copy of the curve.

2. Triangle ABC has vertices $A = (-1,-1,-4)$, $B = (0,-2,-2)$, and $C = (-1,-3,-2)$ in $\mathbb{R}^3$. What is the angle at vertex B?

3. Consider

   - $x = \frac{1}{2}t - \frac{\sqrt{3}}{2}\sin t$ and $y = \frac{\sqrt{3}}{2}t + \frac{1}{2}\sin t$ for $-\pi \leqslant t \leqslant \pi$,
   - $y = x\sqrt{3}$.

   Find the area of the region of the plane enclosed by these two curves.