**Fall 2025** **Math 53, Sections 104/106**

# Discussion #1

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1. Find Cartesian equations $f(x, y) = 0$ for the following parametrized curves.

   (a) $x = \sqrt{t+1}$, $y = \frac{1}{t+1}$, for $t > -1$.

   (b) $x = 4 - 2t$, $y = 3 + 6t - 4t^2$.

   (c) $x = 2e^t$, $y = \cos(1 + e^{3t})$.

2. Compute the slopes of the following curves at a point in time $t$. Find the points where the tangents are vertical and horizontal and compute the second derivative $d^2y/dx^2$ at the horizontal points.

   (a) $x = \cos t, y = \sin t$.

   (b) $x = t^2 - 1$ $y = t^3 - t$.

   (c) $x = e^t - 1, y = \sin t$.

3. Find the length of each curve.

   (a) $x = 1 + 3t^2$, $y = 4 + 2t^3$, $0 \leqslant t \leqslant 1$.

   (b) $x = e^t - t$, $y = 4e^{t/2}$, $0 \leqslant t \leqslant 2$.

   (c) $x = e^t \cos t$, $y = e^t \sin t$, $0 \leqslant t \leqslant \pi$.

4. Convert the following from polar to Cartesian, or vice versa. What curve does it trace out?

   (a) $x + y = 1$, $0 \leqslant y \leqslant 1$.

   (b) $r = -3$, $0 \leqslant \theta \leqslant \pi$.

5. Compute the slopes of the following curves. Find the points where the tangents are vertical and horizontal.

   (a) $r = 3\cos\theta$;

   (b) $r = 1 - \sin\theta$;

   (c) $r = \sec\theta$;

   (d) $r = e^\theta$.