Arithmetic Geometry and Number Theory RTG Seminar

Organizer: Martin Olsson, Sug Woo Shin, Xinyi Yuan

Monday, 3:10–5:00 pm, 748 Evans Hall

Apr 2 **Martin Weissman**, UCSC

Extending the Langlands program to covering groups.

Title (re-talk): An introduction to metaplectic groups

Abstract (re-talk): In his 1964 Acta paper, André Weil introduced metaplectic groups. For Weil, these were groups generated by certain unitary operators on a space of L^2 functions. His paper brought together harmonic analysis and number theory, yielding new results on quadratic forms and a proof of quadratic reciprocity. Within about 10 years, Shimura had carried out a deep study of modular forms of half-integer weight and Gelbart and Piatetski-Shapiro linked half-integer weight modular forms to automorphic forms on Weil's metaplectic groups.

The metaplectic groups are central extensions of symplectic groups by a group of order 2. Soon after Weil's analytic construction, Steinberg and Matsumoto studied central extensions of Chevalley groups (like $SL_n(F)$) over arbitrary fields, finding a link to algebraic K-theory. This provided an algebraic approach to metaplectic groups, and a broader class of groups to study with applications to automorphic forms and number theory.

In this talk, I will give a historical introduction to the metaplectic group and its generalizations, focusing on algebraic aspects and motivating Brylinski and Deligne's category of "central extensions of reductive groups by K2". No familiarity with algebraic groups, K-theory, or automorphic forms is required. The style will be comparable to a "What is..." paper in the Notices.

Title (advanced talk): Extending the Langlands program to covering groups.

Abstract (advanced talk): Among the very first modular forms, studied by Jacobi, were modular forms of half-integer weight. In modern terms, these can be viewed as automorphic forms on the metaplectic group. Since the metaplectic group is not an algebraic group, the conjectures of Langlands do not apply – Langlands did not conjecture a relationship between automorphic representations of metaplectic groups and Galois representations, for example.

I will describe recent efforts to close this basic gap in the Langlands program, by constructing an "L-group" for a broad class of covering groups (including the metaplectic group). This L-group allows one to reformulate Langlands' conjectures for covering groups. I will discuss the classification of covering groups (after Brylinski-Deligne), the construction of this L-group, and evidence for an extension of the Langlands program.