MATH 1A MIDTERM II, XINYI YUAN
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(5 PAGES)

Problem Number	1	2	3	4	5	Total
Score						

YOUR NAME: ____________________

Your GSI's Last Name (please circle): Ballus Armet, Bertoloni Meli, Farid, Hanlon, Hung, Munteanu, Nayak, Sherman

Time of your discussion section: ______________

1. (7 points) Find the derivative of

$$f(x) = \ln(x + \sqrt{x^2+1}).$$

Simplify your answer as much as possible.

Sol:

$$f'(x) = \frac{1}{x+\sqrt{x^2+1}} \cdot \left(1 + \frac{1}{2}(x^2+1)^{\frac{1}{2}-1} \cdot 2x\right)$$

$$= \frac{1}{x+\sqrt{x^2+1}}\left(1 + \frac{x}{\sqrt{x^2+1}}\right)$$

$$= \frac{\sqrt{x^2+1}+x}{(x+\sqrt{x^2+1})\sqrt{x^2+1}}$$

$$= \frac{1}{\sqrt{x^2+1}}$$

2. (7 points) Assume the relation

$$1 + x^2y^2 = e^{x+y}.$$

Express $\frac{dy}{dx}$ in terms of x and y.

Sol: Take derivatives on both sides of the equation

Get

$$2xy^2 + 2x^2yy' = e^{x+y}(1+y')$$

$$(2x^2y - e^{x+y})y' = e^{x+y} - 2xy^2$$

$$y' = \frac{e^{x+y} - 2xy^2}{2x^2y - e^{x+y}}.$$

3. (8 points) Find the limit

$$\lim_{x \to 0+} x^{\sin x}.$$

Sol: $x^{\sin x} = e^{\sin x \ln x}$

$$\lim_{x \to 0^+} \sin x \ln x$$

$$= \lim_{x \to 0^+} \frac{\ln x}{\frac{1}{\sin x}}$$

$$= \lim_{x \to 0^+} \frac{\frac{1}{x}}{-\frac{1}{\sin^2 x} \cos x}$$

$$= \lim_{x \to 0^+} \left(- \frac{\sin x}{x} \cdot \frac{\sin x}{\cos x} \right)$$

$$= -1 \cdot \frac{0}{1}$$

$$= 0$$

Therefore,

$$\lim_{x \to 0^+} x^{\sin x} = \lim_{x \to 0^+} e^{\sin x \ln x} = e^0 = 1.$$

4. (8 points) Find the absolute maximal value and the absolute minimal value of the function

$$f(x) = x^4 - 4x^3$$

in the interval $[-1, 4]$.

Sol: $f'(x) = 4x^3 - 4 \cdot 3x^2 = 4x^2(x-3)$

critical points: $x=0$, $x=3$.

Compare the values:

$f(0) = 0$

$f(3) = -27$ smallest

$f(-1) = 5$ biggest

$f(4) = 0$

Conclusion:

absolute max $f(-1) = 5$,

absolute min $f(3) = -27$.

5. (10 points) Graph the function

$$f(x) = \frac{1}{x^2 - 1}.$$

You may freely use the following derivative formulae:

$$f'(x) = -\frac{2x}{(x^2-1)^2}, \qquad f''(x) = \frac{2(3x^2+1)}{(x^2-1)^3}.$$

The graph should clearly show symmetry, intercepts, asymptotes, local extremes, concavity and inflection points, if there are any of them. Explain how you get these properties. (Pure plotting will not receive full credit. Please write on the back of this page if more space is needed.)

Sol: (1). Domain: $x^2 - 1 \neq 0$, $x \neq \pm 1$

$(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

(2). Symmetry: $f(x) = f(-x)$, even.

(3). intercepts: $(0, -1)$

(4). horizontal asymptotes: $y = 0$ both direc.

$$\lim_{x \to \infty} f(x) = \lim_{x \to -\infty} f(x) = 0.$$

(5). Vertical asymptotes: $x = -1$, $x = 1$

$$\lim_{x \to (-1)^-} f(x) = \infty, \quad \lim_{x \to (-1)^+} f(x) = -\infty.$$

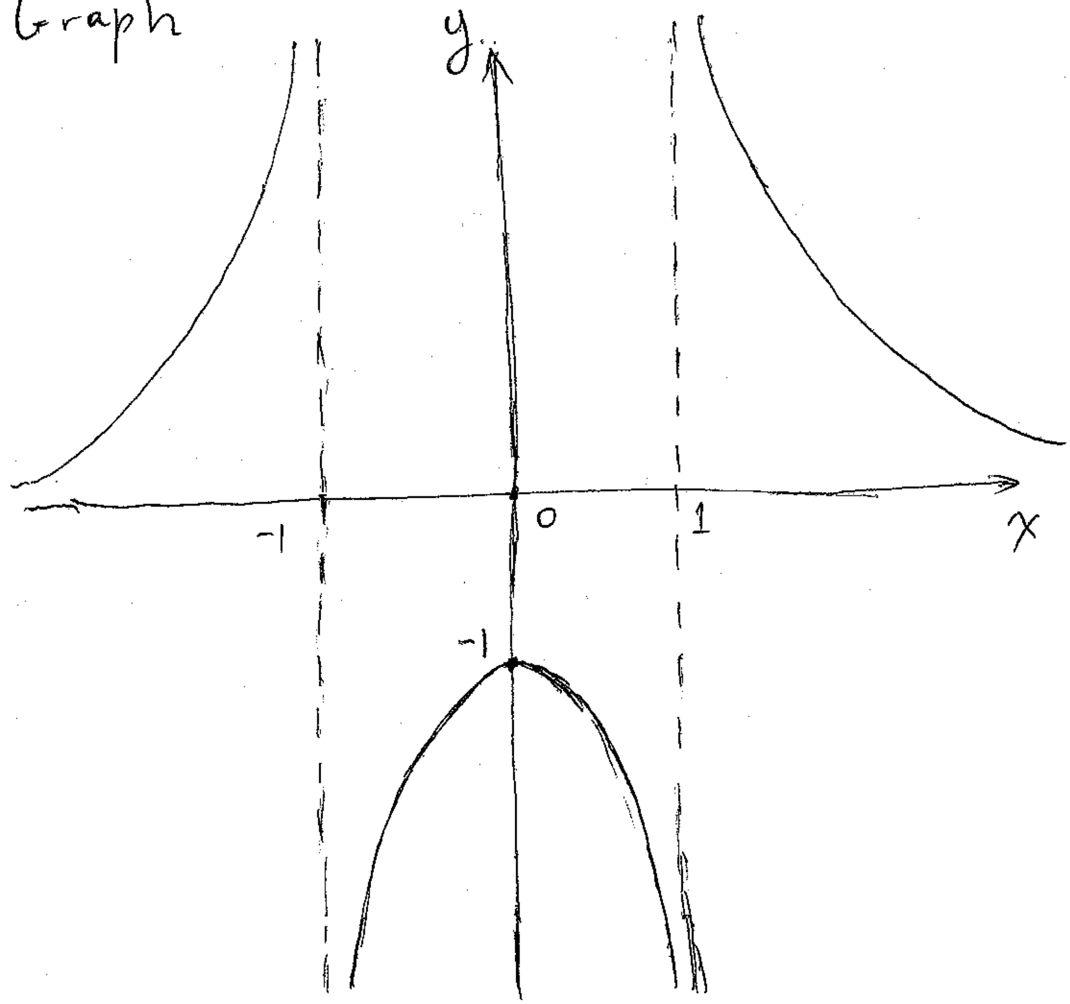
$$\lim_{x \to 1^-} f(x) = -\infty, \quad \lim_{x \to 1^+} f(x) = \infty.$$

(6). Critical pts: $(0, 0)$ from $f'(x) = 0$.

(7). Increasing / Decreasing and concavity

	$(-\infty, -1)$	$(-1, 0)$	$(0, 1)$	$(1, \infty)$
$f'(x)$	$+$	$+$	$-$	$-$
$f''(x)$	$+$	$-$	$-$	$+$
Inc/Dec	$\nearrow$	$\nearrow$	$\searrow$	$\searrow$
Concavity	$\smile$	$\frown$	$\frown$	$\smile$

(8). Graph



There is no inflection point by the graph.