MATH 1A MIDTERM I, XINYI YUAN
12:10PM-12:50PM, SEPT. 22, 2014
(6 PAGES)

Problem Number	1	2	3	4	5	6	Total
Score							

YOUR NAME: ______________________

Your GSI's Last Name (please circle): Ballus Armet, Bertoloni Meli, Farid, Hanlon, Hung, Munteanu, Nayak, Sherman

Time of your discussion section: ______________

1. (6 points) Find the domain of the function
$$f(x) = \ln(-x^{-1}\ln\ln x).$$

Sol: x^{-1} defined $\Rightarrow x \neq 0$

$\ln x$ defined $\Rightarrow x > 0$

$\ln\ln x$ defined $\Rightarrow \ln x > 0 \Rightarrow x > 1$

$\ln(-x^{-1}\ln\ln x)$ defined $\Rightarrow -x^{-1}\ln\ln x > 0$

$\Rightarrow \ln\ln x < 0$ (since $x > 0$)

$\Rightarrow \ln x < 1$

$\Rightarrow x < e$

Conclusion: $1 < x < e$, or write $(1, e)$.

2. (10 points) Simplify the following expressions:

(1) $\cos(\tan^{-1}\frac{3}{4})$.

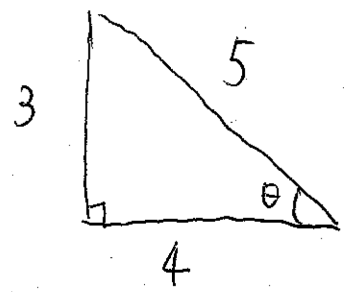
(2) $\sin^{-1}(\frac{1}{8})+\cos^{-1}(\frac{1}{8})$.

(Here $\sin^{-1}, \cos^{-1}, \tan^{-1}$ denote the inverse trigonometric functions.)

Sol: (1). $\tan\theta = \frac{3}{4}$

$\cos\theta = \frac{4}{5}$

by the triangle

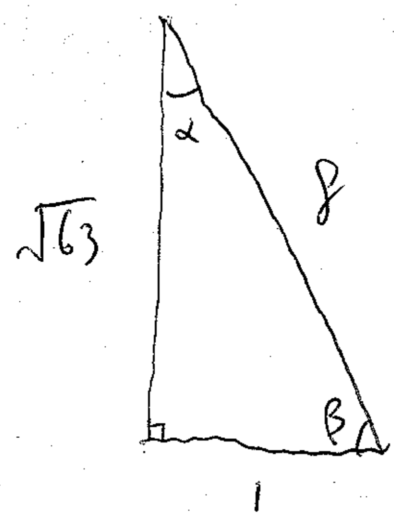


(2). In the triangle,

$\alpha = \sin^{-1}(\frac{1}{8})$

$\beta = \cos^{-1}(\frac{1}{8})$

$\alpha + \beta = \frac{1}{2}\pi$



Therefore,

$$\sin^{-1}(\frac{1}{8}) + \cos^{-1}(\frac{1}{8}) = \frac{\pi}{2}.$$

3. (6 points) Graph the function

$$f(x) = \frac{x}{2-x}.$$

(You do not have to use accurate scales, but you need to make the x-axis, y-axis and the asymptotes clear. Explain briefly how you get your graph.)

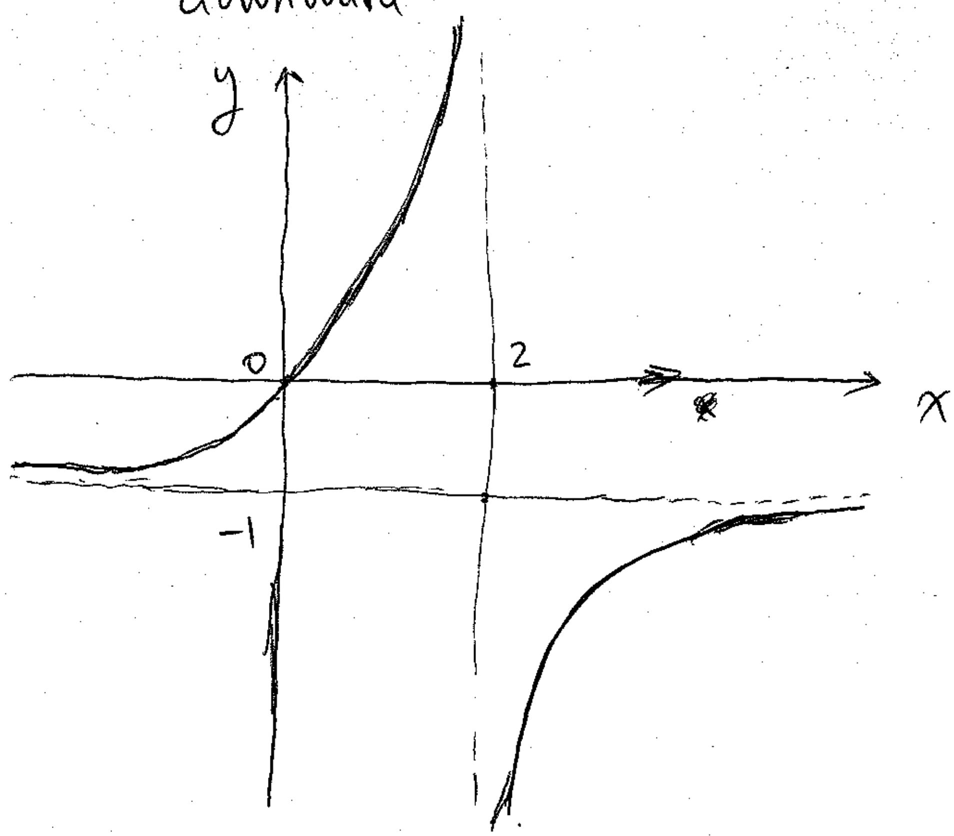
Sol: $f(x) = \left(\frac{x}{2-x} + 1\right) - 1 = \frac{2}{2-x} - 1$

Graph of f is obtain by

(1). Stretch the graph of $-\frac{1}{x}$ by 2 times in the vertical direction.

(2). Shift to the right by 2

(3). Shift ~~upward~~ downward by 1



4. (6 points) Compute the limit

$$\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{x^2 - x} \right).$$

(The answer could be an explicit finite number, ∞, $-\infty$, or none of these three cases. Justify your answer.)

Sol: $\frac{1}{x} - \frac{1}{x^2 - x}$

$$= \frac{x-2}{x(x-1)}$$

As $x \to 0$, have $\frac{x-2}{x-1} \to \frac{-2}{-1} = 2$

But $\lim_{x \to 0^+} \frac{1}{x} = \infty$, $\lim_{x \to 0^-} \frac{1}{x} = -\infty$.

Then

$$\lim_{x \to 0^+} \frac{1}{x} = \infty,$$

$$\lim_{x \to 0^-} \frac{1}{x} = -\infty.$$

Conclusion: DNE (none of the three types).

5. (6 points) Compute the limit

$$\lim_{x\to 4} \frac{\sqrt{x}-2}{x^2-7x+12}.$$

(The answer could be an explicit finite number, ∞, $-\infty$, or none of these three cases. Justify your answer.)

Sol:

$$\frac{\sqrt{x}-2}{x^2-7x+12}$$

$$= \frac{\sqrt{x}-2}{(x-4)(x-3)}$$

$$= \frac{(\sqrt{x}-2)(\sqrt{x}+2)}{(x-4)(x-3)(\sqrt{x}+2)}$$

$$= \frac{x-4}{(x-4)(x-3)(\sqrt{x}+2)}$$

$$= \frac{1}{(x-3)(\sqrt{x}+2)}$$

The limit $= \frac{1}{(4-3)(\sqrt{4}+2)} = \frac{1}{4}$.

6. (6 points) Compute the limit

$$\lim_{x\to 0} x^2\left(\sin\left(\frac{x-2}{x^3+x^2}\right)+\frac{x-2}{x^3+x^2}e^{x+3}\right).$$

(The answer could be an explicit finite number, ∞, $-\infty$, or none of these three cases. Justify your answer.)

Sol: By the distribution law, write

$$x^2\left(\sin\frac{x-2}{x^3+x^2}+\frac{x-2}{x^3+x^2}e^{x+3}\right)$$

$$= x^2\sin\frac{x-2}{x^3+x^2}+\frac{x-2}{x+1}e^{x+3}$$

Note that

$$-x^2 \le x^2\sin\frac{x-2}{x^3+x^2} \le x^2$$

and $\lim\limits_{x\to 0}(x^2) = \lim\limits_{x\to 0}(-x^2) = 0$

By the squeeze thm,

$$\lim_{x\to 0} x^2\sin\frac{x-2}{x^3+x^2} = 0.$$

On the other hand,

$$\lim_{x\to 0}\frac{x-2}{x+1}e^{x+3} = \frac{0-2}{0+1}e^{0+3} = -2e^3.$$

Therefore, $\lim\limits_{x\to 0} x^2\left(\sin\left(\frac{x-2}{x^3+x^2}\right)+\frac{x-2}{x^3+x^2}e^{x+3}\right) = 0-2e^3$
$= -2e^3$