MATH 1A FINAL EXAM SAMPLE, XINYI YUAN, FALL 2014

1. Find the following limits:

$$\lim_{x\to 1}\frac{x^3-1}{x^2-1}, \qquad \lim_{x\to 0} x^{-2}(e^x+e^{-x}-2).$$

2. Find the derivatives of the following functions:

$$f(x)=\cos(x)\ln\sin(x^2), \qquad g(x)=\int_{x^2}^{x^3}\sin(t^2)dt.$$

3. Compute the following integrals:

$$\int \frac{x^3}{\sqrt{x^2+1}}dx, \qquad \int_1^2 \frac{1}{x}(\ln x+x^2)dx.$$

4. Find the maximal value and the minimal value of the function

$$f(x)=xe^{-\frac{x^2}{8}}$$

on the interval $[-1,4]$.

5. Compute the area of the region in the xy-plane bounded by the graphs of

$$y=x, \quad y=1-\frac{1}{2}x, \quad y=\frac{1}{2}x^2,$$

and lying below the lines $y=x$ and $y=1-\frac{1}{2}x$.

6. Let R be the region in the previous problem, i.e., the region in the xy-plane bounded by the graphs of

$$y=x, \quad y=1-\frac{1}{2}x, \quad y=\frac{1}{2}x^2,$$

and lying below the lines $y=x$ and $y=1-\frac{1}{2}x$. Compute the volume of the solid obtained by rotating R about the x-axis.

7. Let S be the region in the xy-plane bounded by the graphs of the equations

$$y=x^3-4x+4, \quad y=0, \quad x=1, \quad x=3.$$

Compute the volume of the solid obtained by rotating S about the y-axis.