INTRODUCTION TO THE LANGLANDS PROGRAM  
(NUMBER THEORY SEMINAR, BERKELEY, FALL 2017)

XINYI YUAN

The goal of this series of talks is to introduce the Langlands program. The main reference of this series is the book [BG].

Lecture 1. Introduction and organization, Xinyi Yuan, Sept 6. I will outline the materials and distribute the weekly lectures to volunteers.

Lecture 2. Artin L-functions. This is a motivation of the Langlands program. Following [BG, §4], the goal of this talk is to introduce Artin L-functions of Galois representations, and Artin’s conjecture about analytic behavior of the L-functions. Sketch a proof of Theorem 4.1 and state Conjecture 5.1 specifically.

Lecture 3. Class field theory and Tate thesis. This is the simplest case of the Langlands program. The goal of this section is to sketch a proof of Artin’s conjecture for 1-dimensional Galois representations via the class field theory and Tate’s thesis. Follow [BG, §6] and [CF, VII, XV].

Lecture 4. L-functions of elliptic curves and modular forms. This is the most important example of the Langlands program. Following [BG, §5], introduce the Eichler–Shimura correspondence between modular forms and elliptic curves, and the modularity theorem of Wiles et al. In particular, explain why the L-functions are equal under the correspondence. A complete account is in [DS].

Lecture 5. From modular forms to automorphic representations. Following [BG, §7], introduce automorphic forms and automorphic representations of GL(2), and the process from a classical modular form (resp. eigenform) to an automorphic form (resp. representation).

Lecture 6. Analytic Theory of L-Functions for GL(n). Following [BG, §9], sketch a proof of the analytic continuation and functional equation for automorphic cuspidal representations of GL(n) over a number field.

Lecture 7. Langlands conjectures I: GL(n). Following [BG, §10], introduce the Langlands conjectures for GL(n), for both the local case and the global case.

Lecture 8. Langlands conjectures II: functoriality. Following [BG, §11], introduce Langlands’s functoriality conjecture for automorphic representations of different groups. List various examples of the functoriality.
Lecture 9. *Satake isomorphisms*. Following [Gr], introduce the Satake isomorphism for the spherical Hecke algebra of a split reductive group over a local field, and especially its consequences on unramified representations of the group.

Lecture 10. *Jacquet–Langlands correspondence*. The goal of this section is to introduce the Jacquet–Langlands correspondence, which is an example of the functoriality. State [Ge, §10, Theorem 10.5], and sketch the proof using the trace formula.

**References**


