INTRODUCTION TO THE LANGLANDS PROGRAM (NUMBER THEORY SEMINAR, BERKELEY, FALL 2017)

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The goal of this series of talks is to introduce the Langlands program. The main reference of this series is the book [BG].

Lecture 1. *Introduction and organization*, Xinyi Yuan, Sept 6. I will outline the materials and distribute the weekly lectures to volunteers.

Lecture 2. *Artin L-functions*. This is a motivation of the Langlands program. Following [BG, §4], the goal of this talk is to introduction Artin L-functions of Galois representations, and Artin's conjecture about analytic behavior of the L-functions. Sketch a proof of Theorem 4.1 and state Conjecture 5.1 specifically.

Lecture 3. *Class field theory and Tate thesis*. This is the simplest case of the Langlands program. The goal of this section is to sketch a proof of Artin's conjecture for 1-dimensional Galois representations via the class field theory and Tate's thesis. Follow [BG, §6] and [CF, VII, XV].

Lecture 4. *L-functions of elliptic curves and modular forms*. This is the most important example of the Langlands program. Following [BG, §5], introduce the Eichler–Shimura correspondence between modular forms and elliptic curves, and the modularity theorem of Wiles et al. In particular, explain why the L-functions are equal under the correspondence. A complete account is in [DS].

Lecture 5. *From modular forms to automorphic representations*. Following [BG, §7], introduce automorphic forms and automorphic representations of GL(2), and the process from a classical modular form (resp. eigenform) to an automorphic from (resp. representation).

Lecture 6. *Analytic Theory of L-Functions for* $\mathrm{GL}(n)$. Following [BG, §9], sketch a proof of the analytic continuation and functional equation for automorphic cuspidal representations of $\mathrm{GL}(n)$ over a number field.

Lecture 7. *Langlands conjectures I:* $\mathrm{GL}(n)$. Following [BG, §10], introduce the Langlands conjectures for $\mathrm{GL}(n)$, for both the local case and the global case.

Lecture 8. *Langlands conjectures II: functoriality*. Following [BG, §11], introduce Langlands's functoriality conjecture for automorphic representations of different groups. List various examples of the functoriality.

Lecture 9. *Satake isomorphisms.* Following [Gr], introduce the Satake isomorphism for the spherical Hecke algebra of a split reductive group over a local field, and especially its consequences on unramified representations of the group.

Lecture 10. *Jacquet–Langlands correspondence.* The goal of this section is to introduce the Jacquet–Langlands correspondence, which is an example of the functoriality. State [Ge, §10, Theorem 10.5], and sketch the proof using the trace formula.

References

[BG] J. Bernstein and S. Gelbart. An Introduction to the Langlands Program. Birkhäuser, 2003.

[CF] J.W.S. Cassels, A. Fröhlich, (Eds), Algebraic Number Theory, Academic Press, 1967.

[DS] F, Diamond, J. Shurman, A first course in modular forms, Springer, New York, 2005.

[Ge] S.S. Gelbart. Automorphic Forms on Adele Groups. Annals of Mathematics Studies, Princeton University Press, 1975.

[Gr] B. H. Gross. On the Satake isomorphism. In Galois representations in arithmetic algebraic geometry (Durham, 1996), volume 254 of London Math. Soc. Lecture Note Ser., pages 223237. Cambridge Univ. Press, Cambridge, 1998.